Mathematics Samples

Answer each of the following questions, showing all working:

- **ARITHMETICS**

1. Find $2 \times 6 - 3$ and $2 \times (6 - 3)$

2. What is the highest common factor of 40 and 15 ?

3. A machine has 3 wheels, making 15, 8 and 22 revolutions (respectively) per minute. At time 0, each of the wheels starts with a certain marked point on its circumference pointing directly downwards. At what time will all of the wheels first come back together in the same starting position?

4. Is 42 a prime number? Why?

5. (**1**) Write 72 as the product of prime factors.
(**2**) Write 26 as the product of prime factors.
(**3**) Are 72 and 26 relatively prime?
(**4**) Write 1872 as the product of prime factors. (Hint: $1872 = 72 \times 26$).

6. Write $\sqrt{200}$ in simplest form.

7. Find $\frac{1}{16} \times \frac{0}{20}$

8. Find $\frac{15}{4} \div \frac{16}{14}$

9. Find $\frac{13}{10} + \frac{-6}{11}$

10. Find $\frac{15}{4} - \frac{15}{4}$

11. Find $\frac{-10}{-8} \times \frac{-16}{43} \div \left(\frac{-16}{-52} \div \frac{-10}{13}\right)$

12. Evaluate $(-3)^2$.

13. Find $|-41 + 34|$

- **ALGEBRA**

1. Find z if $\sqrt{50} = z\sqrt{2}$

2. Write $x \geq 6.0$ in interval form and mark it on a real line.

3. Write the interval $(-\infty, 18.5]$ using an inequality sign and mark it on a real line.

4. Find y if $\sqrt{2197y} = 13\sqrt{13}$

5. Find y, if $-1 = 4y$

6. Find z, if $6z - 3 = 6$

7. Expand $(1 - 4z)(-5z)$

8. Expand $(1 + 2z)(1 + 2z)$

9. Find z, if $|2z + 1| = 6$

10. Expand and simplify $\left(\sqrt{6} + \sqrt{6}\right)\left(\sqrt{6} + \sqrt{6}\right)$

11. Let $y = 1$. Find z, if $2z + 2 = y$

12. Expand and simplify $\sqrt{5}\left(\sqrt{2} + \sqrt{3}\right)$

13. Find x, if $\frac{4x}{4} + 6 = 4$

14. Find z, if $\frac{4}{4z} + 5 = 6$

15. Find x, if $x = \frac{2}{-7} \div \frac{4}{3}$

16. Simplify $x^{-2}x^{-3}y^{2}y^{-2} \times y^{-2} \div x^{0}$

17. Solve $-x - 4 \leq 3x - 20$, then write your answer in interval format and mark it on a real line.

18. Simplify $\frac{15x^{-2}x^{-1}}{x^{-3}x^{-3}}$

19. What two consecutive integers add together to give 17?

20. What two integers that differ by 5 add together to give 17?

- **SIGMA NOTATION**

1. Find z if $\sum_{i=4}^{7} 2z = -32$

2. Find z if $\sum_{i=-2}^{0} zi = -6$

3. Evaluate $\sum_{k=2}^{6} (-2)^{k}\, k$

4. Expand and simplify $\sum_{j=-2}^{6} -4jz$

5. Find x if $x = \sum_{j=2}^{5} 4j^{2}$

6. Write in summation notation: $-\frac{1}{4} - \frac{1}{5} - \frac{1}{6} - \frac{1}{7} - \frac{1}{8} - \frac{1}{9}$

7. Find z if $\sum_{j=z-1}^{z} j = 5$

- **SETS**

1. Given two sets:
$A = \{-3, 5, 0, 8, 6\}$ and $G = \{3, -1, 5, 1, 6\}$,
find $A \cap G$.
Illustrate your answer with Venn diagram.

2. Given two sets $E = \{-3, 0, 9\}$ and $F = \{3, 5, 2, 0, 9, 8, 1, 6\}$,
find:

(**1**) $E \cap F$
(**2**) $E \cup F$
(**3**) $E \backslash F$
(**4**) $F \backslash E$

Illustrate your answers with Venn diagrams.

3. For the following questions let $F = \{-1, 0, 4, 8, 6, 1\}$, $G = \{3, -1, 7, 9, 4, 6\}$, $H = \{5, 2, 8\}$

(**1**) Write down the elements of set F .
(**2**) Write down the elements of the set $F \cup H$.
(**3**) Write down the elements of the set $G \cap F$.
(**4**) Write down the elements of the set $H \backslash F$.
(**5**) Write down the elements of the set $G \backslash (F \cup H)$. Shade the corresponding region on the Venn diagram.

(**6**) Write down the elements of the set $(G\backslash H)\backslash F$.
(**7**) Write down the elements of the set $H \cup (F \cap G)$.
(**8**) Write down the elements of the set $\emptyset \cup H$.
(**9**) Write down the elements of the set $(F\backslash G) \cup (G\backslash H)$.

4. For the following questions let s_1 and s_2 be random natural numbers chosen independently, where s_1 is between 5 and 9 (inclusive), and s_2 is between 1 and 4 (inclusive). In each case, find the probability p that:

(**1**) s_1 is even?
(**2**) $s_1 = 8$?
(**3**) $s_1 \geq 8$?
(**4**) s_1 is even and $s_1 \geq 8$?
(**5**) s_1 is even or $s_1 \geq 8$?
(**6**) s_1 is even given that $s_1 \geq 8$?
(**7**) Both s_1 and s_2 are even ?
(**8**) At least one of s_1 and s_2 is even ?
(**9**) s_1 is even given that s_2 is odd ?

- **STRAIGHT LINES AND SIMULTANEOUS EQUATIONS**

1. Find the distance between the points $(-6, 9)$ and $(-2, -10)$.
2. Find the gradient and y-intercept of the line $1 + 10x = -7y$.
3. Find the gradient and y-intercept of the line $2 + 3x + 4y = 8y + 7 - 7x$.
4. Find the equation of the straight line with gradient $m = 2$ passing through the point $(0, -9)$.
5. Find the equation of the straight line passing through the points $(-7, 7)$ and $(9, 0)$.
6. Find the equation of the line parallel to $-15x = 5y + 20$ and passing through the point $(-4, 10)$.
7. Find the equation of the line parallel to $6y - 4 + 9x = 5y - 10 + 13x$ and passing through the point $(0, 8)$.
8. Find the equation of the line perpendicular to $6y = 18x - 36$ and passing through the point $(-27, 19)$.
9. Does the line $70x = -7y - 63$ pass through the point $(-2, 11)$?
10. Find the equation of the line perpendicular to $21 = 3y$ and passing through the point $(-2, -9)$.
11. Find the equation of the line perpendicular to $5x = 4$ and passing through the point $(-7, 0)$.
12. Find the equation of the line parallel to $-5 = -5x$ and passing through the point $(0, 9)$.
13. Find the equation of the line parallel to $0 = 70 + 10y$ and passing through the point $(2, 8)$.
14. Solve

$$\begin{aligned} -9y &= -4x - 540 \\ 60x - 20 &= -10y \end{aligned}$$

15. Solve

$$\begin{aligned} 2y - 6x &= -6 \\ 6y - 18x &= -22 \end{aligned}$$

16. Solve

$$\begin{aligned} 3y - 5\tan x &= -11 \\ 3y - 10\tan x &= -16 \end{aligned}$$

given $0 \leq x < 2\pi$

17. Do the lines $-8y - 7x = 12$ and $-2y + 5x = 30$ intersect? If so, find the point of intersection.

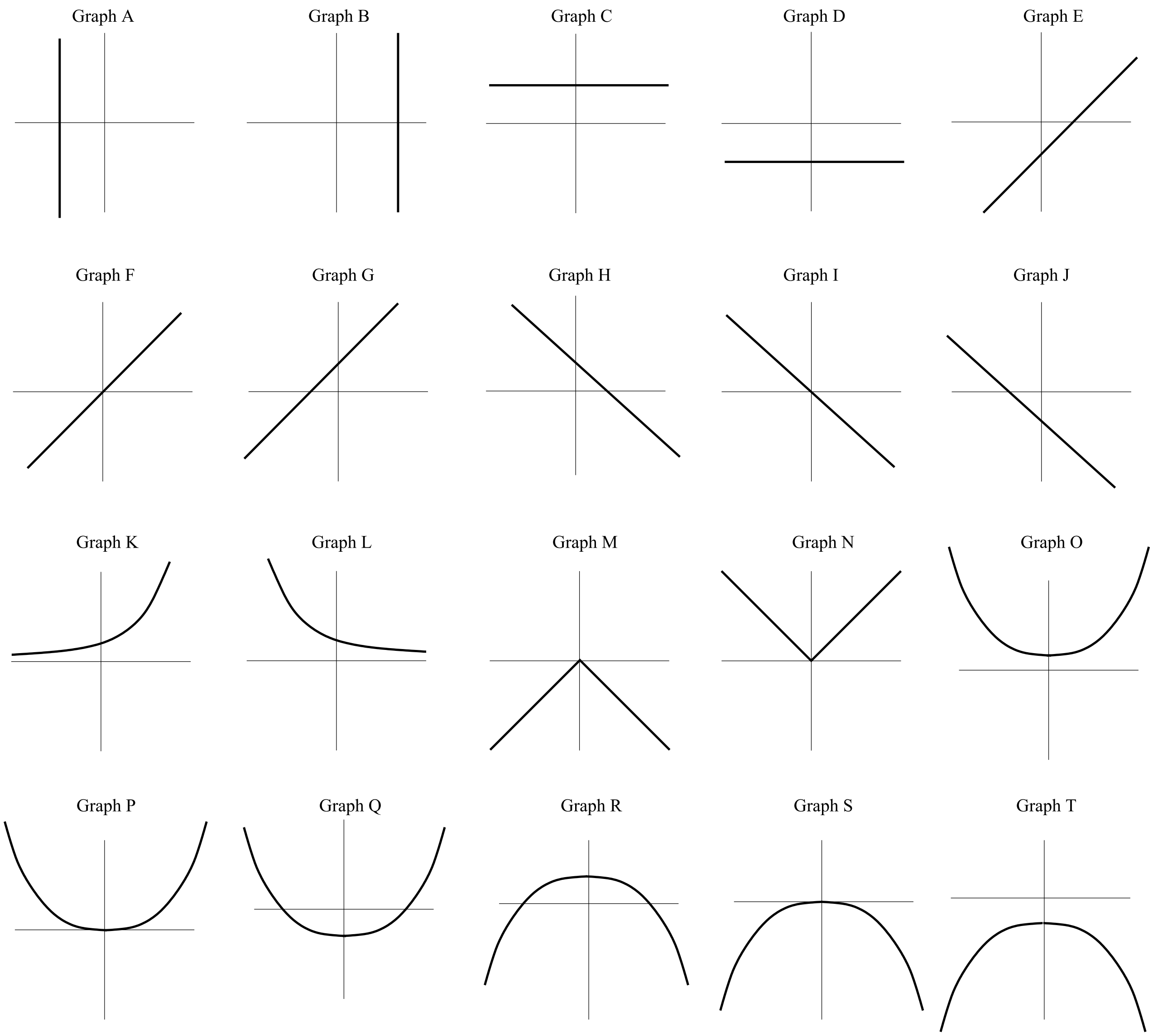


Figure 1: Graphs of various equations.

18. Do the lines $100y + 10x + 91 = 0$ and $-6x - 54 - 60y = 0$ intersect? If so, find the point of intersection.

- **FUNCTIONS, QUADRATICS AND TRIGONOMETRIC FUNCTIONS**

1. There are eight equations given in this question and you need to match each equation with its corresponding graph. The graphs are shown in Figure 1.

(1) $y = 2 \times |-7x|$
(2) $y = e^{-7x}$
(3) $x = -15y - 12x$
(4) $-9 = -y + 9x$
(5) $y - 10x - 8 = 3y - 13x - 8$
(6) $7y + 14x^2 = 12y + 15x^2$
(7) $y = e^{2x}$
(8) $0 = 13y + 10x + 12$

2. Find the domain of $f(x) = \left(\left|\frac{8}{x}\right|\right)^2$.

3. Find the domain of $f(w) = \frac{-6}{|w|+2}$.

4. Find the range of $f(x) = \frac{7}{(-4x)^2}$.

5. Find the range of $f(x) = \frac{1}{12-6x}$.

6. Find the domain and the range of $f(w) = \left|\frac{-9}{-8+w}\right|$.

7. Solve $3y^2 - 7y = 12 + 2y$.

8. Solve each of the following equations **without** using the quadratic formula:

(1) $-6z(-8+9z) = 0$

(2) $(-4z+10)(4+7z) = 0$

(3) $(4x+5)(-9+x) = 0$

(4) $(3y+8)^6 = 0$

9. Solve $(1-2z)^8 = 0$.

10. Find $f(3)$ where $f(x) = 5x^2 + 4x - 8$.

11. On a set of axes sketch the graphs of $y = \sin x$ and $y_1 = -\sin(2x)$ for $x \in [-2\pi, 2\pi]$.

12. Convert each of the following angles from radians to degrees:

$$-\frac{4\pi}{5} \quad -\frac{5\pi}{4} \quad \frac{7\pi}{20} \quad -\frac{23\pi}{12} \quad -\frac{3\pi}{20} \quad \frac{23\pi}{12} \quad -\frac{21\pi}{20} \quad -\frac{11\pi}{3}$$

13. Convert each of the following angles from degrees to radians:

$$-200^\circ \quad -378^\circ \quad 180^\circ \quad 135^\circ \quad -216^\circ \quad -180^\circ \quad -300^\circ \quad -1080^\circ$$

• LOGARITHMS AND EXPONENTIALS

1. Without using a calculator, find each of:

(1) $\log_{12} 12^{12}$

(2) $\log_3 3$

(3) $\log_5 \frac{1}{125}$

(4) $\log_{10} 10$

(5) $\log_{10} \frac{1}{1000000}$

(6) $\ln 1$

(7) $\ln \frac{1}{e^3}$

(8) $\log_{16} 4$

2. Populations of bacteria (and indeed many other organisms) exhibit population growth that can be modelled using continuous compounding. The rate of population growth depends on such factors as fertility, temperature, level of interactions and so on.

A certain type of bacterium has a population growth rate of 0.02 per hour. If there are 800 million bacteria in a petri dish at time 0, how many will there be after 6 hours? (Give your answer in millions, rounded to two decimal places.)

3. 0.10 of a certain radioactive substance decays every 1000 years. If the initial quantity of the substance is 100 units, how many units will remain after 3 thousand years? (Round your answer to two decimal places.)

4. If \$400 is invested for 1 year at a rate of 7.0% per annum, find the final balance if interest compounds:

(1) annually?

(2) every six months?

(3) quarterly?

(4) monthly?

(5) continuously?

5. The doubling time of an exponentially growing population of bacteria is $h = 38.5$ hours. Find its growth rate.

6. A population of bacteria is growing with a growth rate of $k = 0.06$ per hour. Find the doubling time.

7. Peter needs to pay a bill of \$800 in 7 years' time. His bank account earns 8.0 percent interest each year, compounding continuously. How much money does he need to invest now in order to exactly cover his bill in 7 years? Ignore fees and taxes, and round your answer to cents.

8. I invest \$1000 in a bank account earning 6.0 percent interest per year for 7 years, continuously compounding. Ignoring taxes and fees, what is the final account balance?

9. Before Damien can marry Celeste, he must prove to her father that he can save the deposit for a house. He has \$600 to invest, his account pays 8.0 percent interest per annum, compounding continuously, and he needs to have a total of \$1200 before he can get married. He is currently 10 years old. At what age can he get married?

10. Peter the Mathematician is invited to attend the First International Congress of Mathematical Shoemaking to be held in Hawaii in 9 months' time. The problem is that Peter would be required to wear shoes but currently he doesn't have any. Clever Peter has a bank account earning 7.0% p.a., compounding quarterly. How much money does he need to invest now in order to buy a pair of shoes and go to Hawaii, assuming the price of a pair of decent shoes is \$400 ? Ignore fees and taxes, and round your answer appropriately.

- **DERIVATIVES AND INTEGRATION**

1. Find y' where $y = 8$

2. Find y' where $y = 3x^2 + 3$

3. Find y' where $y = -7x^6$

4. Find y' where $y = -\cos x$

5. Find y' where $y = 6e^x$

6. Find y' where $y = -\dfrac{8}{x} + 4\sin x - 7\sqrt{x} + 2\sin x$

7. Let $f(x) = -x^3 - 3x^2 + 9x$.

Q1 Find $f'(x)$.

Q2 Solve $f'(x) = 0$.

Q3 Find $f''(x)$.

Q4 Find $f'(5)$.

8. Find $\dfrac{dy}{dx}$, if $y = \dfrac{1}{(10 + 9x^{-2})^7}$.

9. If $y = \dfrac{9r - 6}{4r + 10}$, find y'.

10. If $y = \dfrac{-9x - 8 - 7x^2}{-1 + 6x^2 - 9x}$, find y'.

11. If $y = \dfrac{-6t^2 + 9}{3t^2 + 10t}$, find y'.

12. Let $y = (8t - 3)\left(6t^3 - 3\right)$. Find y' using the product rule.

13. Find $\int (10x^4 - 7x + 5)\,dx$.

14. Find $\int_{-3}^{0} \left(9x^2 - 8x + 2\right)dx$.

15. Evaluate $\int_{3\pi/2}^{5\pi/2} (-3\sin(-3x))\,dx$

- **MATRICES**

1. Evaluate: $\begin{pmatrix} 0 & 6 & -3 \\ 5 & 6 & -4 \end{pmatrix} + \begin{pmatrix} -6 & -6 & -3 \\ 6 & -1 & -5 \end{pmatrix}$

2. Evaluate: $\begin{pmatrix} 5 & -2 \\ -2 & 3 \\ -2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 5 & 3 \end{pmatrix}$

3. Evaluate: $\begin{pmatrix} -2 & 0 & -5 \\ -4 & -5 & -2 \end{pmatrix} \times \begin{pmatrix} -5 & -2 \\ 2 & 1 \\ -6 & -5 \end{pmatrix}$

4. Let $A = \begin{pmatrix} -3 & 1 \\ -6 & -1 \end{pmatrix}$. Find A^{-1}.

5. Evaluate: $-6 \times \begin{pmatrix} 3 & 3 & -6 \end{pmatrix}$

6. Evaluate: $0\begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} + 3\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$

7. (a) Evaluate

$$\begin{pmatrix} -3 & -6 & -4 \\ 2 & 3 & 3 \\ 3 & -2 & 6 \end{pmatrix}\begin{pmatrix} 24 & 44 & -6 \\ -3 & -6 & 1 \\ -13 & -24 & 3 \end{pmatrix}$$

(b) Use Part (a) to solve the following three simultaneous equations

$$\begin{aligned} -3x - 6y - 4z &= 4 \\ 2x + 3y + 3z &= 1 \\ 3x - 2y + 6z &= -2 \end{aligned}$$

- **BASIC PROGRAMMING WITH PYTHON**

1. Show how to evaluate each of the following numerical expressions in Python, and evaluate the results:

(**1**) $4 \times (6 - 2)^{2\times 4}$

(**2**) $4 - (4 + 2) + 2 - 2$

(**3**) $(4 - 5 \times 5 \times 2) \times 3$

(**4**) $\left(6^{3+6}\right)^{6-5}$

(**5**) $6 + 6^2 \times 4 \times 5$

(6) $3^4 + 3 \times 5^3$
(7) $6^{3+1} - 5 \times 5$
(8) $2 \times (1 - (6 + 4 \times 2))$
(9) $(4 - 5 \times 3) \times 3 + 1$
(10) $(2 - 4)^{4 \times 1^4}$

2. Use Python to calculate the distance between the two points $(0, 3)$ and $(-4, -4)$.

3. Use Python to numerically verify that $\sin^2\theta + \cos^2\theta = 1$, where $\theta = 2$ radians.

4. Use Python to calculate the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$, where $\theta = 4°$. Don't forget that Python uses radians by default, so you will have to convert this value into radians first!

5. Use Python to numerically verify that $\log_{10} 10^x = x$ and $10^{\log_{10} x} = x$ where $x = 12$.

6. Use Python to calculate e^{-3}.

7. Given the following matrix:

$$A = \begin{pmatrix} 0 & 1 \\ 13 & 12 \end{pmatrix}$$

use Python to find its inverse B and show that:

$$AB = BA = I$$

8. Use Python's matrix operations to numerically solve the following set of simultaneous equations:

$$\begin{aligned} 10x + 3y &= 2 \\ -14x + 5y &= 16 \end{aligned}$$

9. Use Python to print out the values of x^3 for all integer values of x from $x = -1$ to $x = -20$ *in that order*.

- **SCIENCE**

1. A particular organism's population is modelled using a simple Leslie model with 2 life stages.
The fertility of each life stage is: for group 1: 3, for group 2: 8.
The survival rate from each life stage to the next is: for group 1: 0.4.
The initial population is: group 1: 10, group 2: 5.
Find the Leslie matrix L and initial population vector P_0, then estimate the population at times $t = 1$ to $t = 3$. (Round your answers to 1 decimal place at each time step.)

2. Suppose we have a 11.2 L sample containing 6.90 mol oxygen gas at a pressure of 2 atm. If all of this oxygen were converted to ozone (O_3) at the same T and P, what would be the volume?

3. A sample of chlorine gas occupies a volume of 504 mL at a pressure of 104.33 kPa. Calculate the pressure of the gas if the volume is increased to 1128 mL (assume temperature remains constant).

4. A sample of diborane gas (B_2H_6) has a pressure of 50.2 kPa at a temperature of 42°C and a volume of 14.90×10^{-3} m^3. If conditions are changed such that the temperature is -12°C and the volume is 13.96×10^{-3} m^3, what will be the pressure of the sample?

5. (1) Ben wants to build his own surfboard. He is going to use foam that has a density of 18 kg/m^3. Using Archimede's principle, what's the minimum mass of foam required to *two decimal places* if Ben has a mass of 100 kg (i.e. the foam is just at the surface of the water: remove any foam and the block will sink)?

(2) Ben has worked out that for his surfboard to work properly, he has to have at least 40% of the block above water. Using the same requirements as above, how much foam does he need now?

Hints:

- 1m^3 of foam will displace 1m^3 of water.
- 1kg of water can support 1kg of another object.
- Water has a density of 1000.0 kg/m^3.

– Ignore the shape of the board: consider the foam to be just a solid block. However, don't forget to take the mass of the block into account!

- **BUSINESS AND FINANCE**

1. Calculate the future value of receiving \$330 at the end of every fortnight for the next 11 years where the discount rate equals 9.1%pa.

2. The present value of receiving an unknown quantity, A, every year for 9 years equals \$465700. Assuming the discount rate equals 8.04% pa, what is the value of A?

3. Given a discount rate of 8.32%pa, how long must we receive \$300 per year for in order for the future value to equal \$800?

4. Calculate the present value of receiving \$178 monthly for 7 years given a discount rate of 11%pa compounded quarterly.

5. Calculate the effective annual rate of 10%pa compounded monthly.

6. *Beta Ltd* (a financial services firm) recently paid a dividend of \$0.10 per share. The remaining 50% of last year's earnings were retained for reinvestment. *Beta Ltd* is not expected to change its dividend policy in the foreseeable future. Assuming that the new investments earn exactly the required return on equity of 8% p.a., at what price do you expect *Beta Ltd* shares be trading?

7. *Gamma Ltd* pays a constant dividend of \$0.10 per share and its shares are currently trading at \$0.50 per share. What is the required equity return for *Gamma Ltd*?

8. *Beta Ltd* has a forward looking P/E ratio of 15 and it has an expected EPS of \$1.10 per share for next year. If the company has a constant dividend payout ratio of 20% of its EPS and the required return on equity is 9%, what is the constant growth of *Beta Ltd*?

9. Consider the utility function, where W is wealth:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

and assume a degree of risk aversion $\gamma = 0.2$. If your current wealth is \$1,010,000, calculate the utility of this level of wealth.

10. Consider the utility function, where W is wealth:

$$U(W) = 1.3 \times W.$$

If your current wealth is \$700,000, and you have the opportunity to play the following game: we toss a fair coin, if it's tails, you win \$381,000; if it's heads, you lose \$381,000. It costs nothing to play this game. What is the expected payoff from the game? And what is the expected utility of your wealth after the game?

11. Consider the utility function, where W is wealth:

$$U(W) = W^{\gamma}$$

and assume $\gamma = 0.7$. If your current wealth is \$55,000, and you have the opportunity to play the following game: we toss a fair coin, if it's tails, you win \$36,000; if it's heads, you lose \$36,000. It costs nothing to play this game. Would you play the game?

12. Suppose that you own a car worth \$41,000 and that it has a 4% probability of being fully destroyed and a 6% probability of losing half of its value in the next year. For simplicity, we assume that the car will retain its value with a 90% probability. Suppose the insurance costs \$1,500 and that you have a logarithmic utility function described by $U(W) = \log_e(W + 100)$. Should you take the insurance?

13. Suppose that you own a car worth \$9,000 and that it has a 4% probability of being fully destroyed and a 6% probability of losing half of its value in the next year. For simplicity, we assume that the car will retain its value with a 90% probability. Assume that you have a logarithmic utility function described by $U(W) = \log_e(W + 500)$. What is the maximum amount of premium you are willing to pay for the insurance?

- **ARITHMETICS**

1. $2 \times 6 - 3 = 12 - 3 = 9$ and $2 \times (6 - 3) = 2 \times 3 = 6$

2. 5

3. The highest common factor of 15, 8 and 22 is 1. This means that in 1 minute (that is, in 60 seconds) the wheels will all come back together to their starting positions.

4. No, since $42 = 2 \times 21$

5. (**1**) $72 = 2 \times 36 = 2 \times 2 \times 18 = 2 \times 2 \times 2 \times 9 = 2 \times 2 \times 2 \times 3 \times 3$
(**2**) $26 = 2 \times 13$
(**3**) No, the highest common factor of 72 and 26 is 2, so **they are not** relatively prime.
(**4**) $1872 = 72 \times 26 = (2 \times 2 \times 2 \times 3 \times 3) \times (2 \times 13) = 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 13$

6. $\sqrt{200} = \sqrt{2 \times 100} = \sqrt{2 \times 2 \times 50} = \sqrt{2 \times 2 \times 2 \times 25}$
$= \sqrt{2 \times 2 \times 2 \times 5 \times 5}$.
Then $\sqrt{200} = 2 \times 5 \times \sqrt{2}$.
Hence the solution is $10\sqrt{2}$

7.

$$\begin{aligned} \frac{1}{16} \times \frac{0}{20} &= \frac{1}{16} \times 0 \\ &= 0 \end{aligned}$$

8.

$$\begin{aligned} \frac{15}{4} \div \frac{16}{14} &= \frac{15}{4} \times \frac{14}{16} \\ &= \frac{15}{4} \times \frac{\not{2} \times 7}{\not{2} \times 8} \\ &= \frac{15}{4} \times \frac{7}{8} \\ &= \frac{15 \times 7}{4 \times 8} \\ &= \frac{105}{32} \\ &= 3\frac{9}{32} \end{aligned}$$

9.

$$\begin{aligned} \frac{13}{10} + \frac{-6}{11} &= \frac{13 \times 11}{10 \times 11} - \frac{6 \times 10}{11 \times 10} \\ &= \frac{143 - 60}{110} \\ &= \frac{83}{110} \end{aligned}$$

10.

$$\frac{15}{4} - \frac{15}{4} = 0$$

11.

$$\begin{aligned}
\frac{-10}{-8} \times \frac{-16}{43} \div \left(\frac{-16}{-52} \div \frac{-10}{13}\right) &= \frac{\cancel{2} \times 5}{\cancel{2} \times \cancel{4}} \times \frac{\cancel{4} \times (-4)}{43} \div \left(\frac{-16}{-52} \div \frac{-10}{13}\right) \\
&= \frac{5}{1} \times \frac{-4}{43} \div \left(\frac{-16}{-52} \div \frac{-10}{13}\right) \\
&= \frac{5 \times (-4)}{1 \times 43} \div \left(\frac{-16}{-52} \div \frac{-10}{13}\right) \\
&= \frac{-20}{43} \div \left(\frac{-16}{-52} \div \frac{-10}{13}\right) \\
&= \frac{-20}{43} \div \left(\frac{16}{52} \times \frac{-13}{10}\right) \\
&= \frac{-20}{43} \div \left(\frac{\cancel{4} \times \cancel{2} \times 2}{\cancel{4} \times \cancel{13}} \times \frac{\cancel{13} \times (-1)}{\cancel{2} \times 5}\right) \\
&= \frac{-20}{43} \div \left(\frac{2}{1} \times \frac{-1}{5}\right) \\
&= \frac{-20}{43} \div \frac{2 \times (-1)}{1 \times 5} \\
&= \frac{-20}{43} \div \frac{-2}{5} \\
&= \frac{-20}{43} \times \frac{-5}{2} \\
&= \frac{\cancel{2} \times (-10)}{43} \times \frac{-5}{\cancel{2}} \\
&= \frac{-10}{43} \times \frac{-5}{1} \\
&= \frac{-10 \times (-5)}{43 \times 1} \\
&= \frac{50}{43} \\
&= 1\frac{7}{43}
\end{aligned}$$

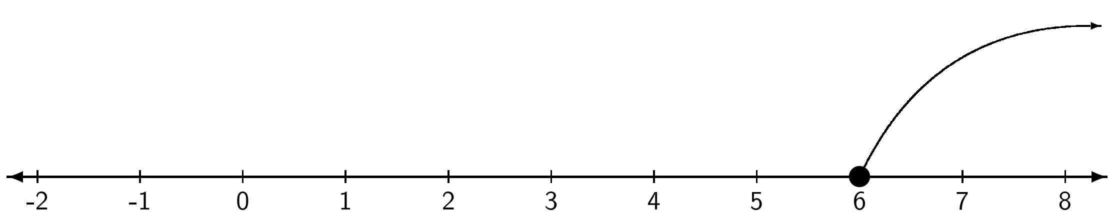
12. $(-3)^2 = -3 \times (-3) = 9$

13. $|-41 + 34| = |-7| = 7$

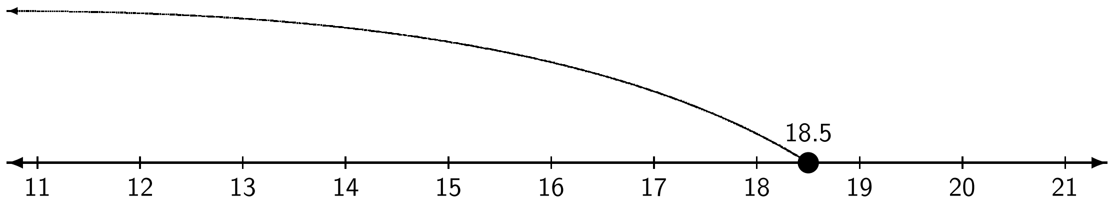
- **ALGEBRA**

1. $\sqrt{50} = z\sqrt{2}$. Now $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{5 \times 5 \times 2} = 5\sqrt{2}$. Hence $z = 5$

2. In interval form the answer is $[6.0, \infty)$ and on a real line the answer is:



3. In inequality form the answer is $x \leq 18.5$ and on a real line the answer is:



4. $\sqrt{2197y} = 13\sqrt{13}$, so $\sqrt{2197y} = \sqrt{13 \times 13 \times 13} = \sqrt{2197}$, so $2197y = 2197$. Hence $y = 1$

5. $-1 = 4y$, so $\frac{-1}{4} = \frac{4y}{4}$

Hence $y = -\frac{1}{4}$

6. $6z - 3 = 6$, so $6z = 6 + 3$, so $6z = 9$, so $\frac{6z}{6} = \frac{9}{6}$

Hence $z = \frac{3}{2}$

7. $(1 - 4z)(-5z) = 1 \times (-5z) - 4z \times (-5z) = -5z + 20z^2$

8. $(1 + 2z)(1 + 2z) = 1 \times 1 + 1 \times 2z + 2z \times 1 + 2z \times 2z = 1 + 2z + 2z + 4z^2 = 4z^2 + 4z + 1$

9. $|2z + 1| = 6$, so

$2z + 1 = 6$	*or*	$2z + 1 = -6$
$2z = 6 - 1$		$2z = -6 - 1$
$2z = 5$		$2z = -7$
$\frac{2z}{2} = \frac{5}{2}$		$\frac{2z}{2} = \frac{-7}{2}$

Hence the solutions are: $z = \frac{5}{2}$ and $z = -\frac{7}{2}$

10.

$$\begin{aligned}\left(\sqrt{6} + \sqrt{6}\right)\left(\sqrt{6} + \sqrt{6}\right) &= \sqrt{6} \times \sqrt{6} + \sqrt{6} \times \sqrt{6} + \sqrt{6} \times \sqrt{6} + \sqrt{6} \times \sqrt{6} \\ &= \sqrt{6 \times 6} + \sqrt{6 \times 6} + \sqrt{6 \times 6} + \sqrt{6 \times 6} \\ &= 6 + 6 + 6 + 6 \\ &= 24\end{aligned}$$

11. Substituting for y into the equation gives $2z + 2 = 1$, so $2z = 1 - 2$, so $2z = -1$, so $\frac{2z}{2} = \frac{-1}{2}$

Hence $z = -\frac{1}{2}$

12.

$$\begin{aligned}\sqrt{5}\left(\sqrt{2} + \sqrt{3}\right) &= \sqrt{5} \times \sqrt{2} + \sqrt{5} \times \sqrt{3} \\ &= \sqrt{5 \times 2} + \sqrt{5 \times 3} \\ &= \sqrt{10} + \sqrt{15}\end{aligned}$$

13. $\frac{4x}{4} + 6 = 4$, so $x = 4 - 6$, so $x = -2$

Hence solution is: $x = -2$

14. $\frac{4}{4z} + 5 = 6$, so $\frac{1}{z} = -5 + 6$, so $\frac{1}{z} = 1$, so $1 = z$

Hence solution is: $z = 1$

15.

$$\begin{aligned}\frac{2}{-7} \div \frac{4}{3} &= \frac{-2}{7} \times \frac{3}{4} \\ &= \frac{\cancel{2} \times (-1)}{7} \times \frac{3}{\cancel{2} \times 2} \\ &= \frac{-1}{7} \times \frac{3}{2} \\ &= \frac{-1 \times 3}{7 \times 2} \\ &= \frac{-3}{14} \\ &= -\frac{3}{14}\end{aligned}$$

Hence solution is: $x{=}-\frac{3}{14}$

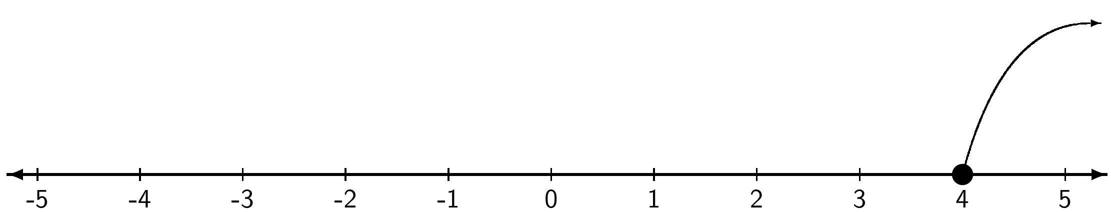
16.

$$\begin{aligned}x^{-2}x^{-3}y^{2}y^{-2} \times y^{-2} \div x^{0} &= x^{-2}x^{-3}y^{2}y^{-2} \times y^{-2} \times x^{0} \\ &= x^{-2}x^{-3}x^{0}y^{2}y^{-2}y^{-2} \\ &= x^{-2-3+0}y^{2-2-2} \\ &= x^{-5}y^{-2}\end{aligned}$$

17.

$$\begin{aligned}-x - 4 &\leq 3x - 20 \\ -x - 4 + 4 &\leq 3x - 20 + 4 \\ -x &\leq 3x - 16 \\ -x - 3x &\leq 3x - 3x - 16 \\ -4x &\leq -16 \\ -4x \div (-4) &\geq -16 \div (-4) \\ x &\geq 4\end{aligned}$$

In interval format the answer is $[4, \infty)$, and on a real line the answer is:



18. $\frac{15x^{-2}x^{-1}}{x^{-3}x^{-3}} = \frac{15x^{-2-1}}{x^{-3-3}} = \frac{15x^{-3}}{x^{-6}} = 15x^{-3-(-6)}{=}\, 15x^{3}$

19. Since the two integers are consecutive, we know that there is a difference of one between them. Let the smaller integer be represented by n, so the larger integer will then be $(n+1)$. We then have:

$$\begin{aligned} & n+(n+1) &=& \ 17 \\ \implies & 2\times n+1 &=& \ 17 \\ \implies & 2\times n &=& \ 16 \\ \implies & n &=& \ 8 \end{aligned}$$

Note that this gives us the value of the *lower* integer only! We need *both* integers!

So if the smaller number is 8, then the larger number must be 9.

20. We know that there is a difference of 5 between the the two integers. Let the smaller integer be represented by n, so the larger integer will then be $(n+5)$. We thus have:

$$\begin{aligned} & n+(n+5) &=& \ 17 \\ \implies & 2\times n+5 &=& \ 17 \\ \implies & 2\times n &=& \ 12 \\ \implies & n &=& \ 6 \end{aligned}$$

Note that this gives us the value of the *lower* integer only! We need *both* integers!

So if the smaller number is 6, then the larger number must be 11.

- **SIGMA NOTATION**

1. $\sum_{i=4}^{7} 2z = -32$, so $2z+2z+2z+2z=-32$, so $8z=-32$

Hence $z=-4$

2. $\sum_{i=-2}^{0} zi = -6$, so $-2z-z+0=-6$, so $-3z=-6$

Hence $z=2$

3. $\sum_{k=2}^{6} (-2)^k\, k = (-2)^2\times 2 + (-2)^3\times 3 + (-2)^4\times 4 + (-2)^5\times 5 + (-2)^6\times 6 = 8-24+64-160+384 = 272$

4. $\sum_{j=-2}^{6} -4jz = 8z+4z+0-4z-8z-12z-16z-20z-24z = -72z$

5. $\sum_{j=2}^{5} 4j^2 = 4\times 2^2 + 4\times 3^2 + 4\times 4^2 + 4\times 5^2 = 16+36+64+100 = 216$

Hence x=216

6. $-\frac{1}{4}-\frac{1}{5}-\frac{1}{6}-\frac{1}{7}-\frac{1}{8}-\frac{1}{9} = \sum_{k=4}^{9}\frac{-1}{k}$

7.

$$\sum_{j=z-1}^{z} j = 5$$

$$z-1+z=5$$

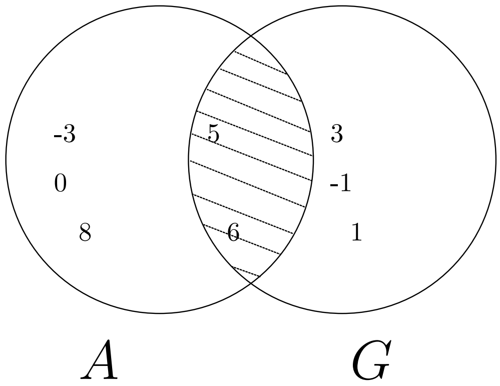
$$2z-1=5$$

$$2z=6$$

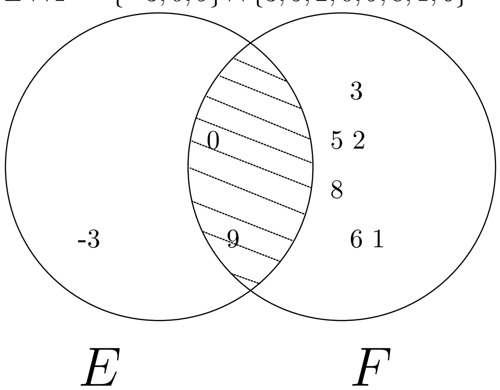
Hence $z=3$

- **SETS**

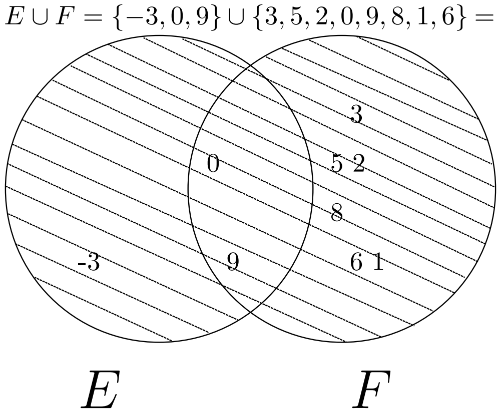
1. $A \cap G = \{-3, 5, 0, 8, 6\} \cap \{3, -1, 5, 1, 6\} = \{5, 6\}$
On Venn diagram:



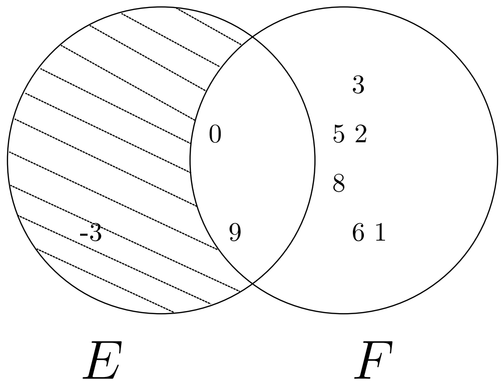
2. **(1)** $E \cap F = \{-3, 0, 9\} \cap \{3, 5, 2, 0, 9, 8, 1, 6\} = \{0, 9\}$



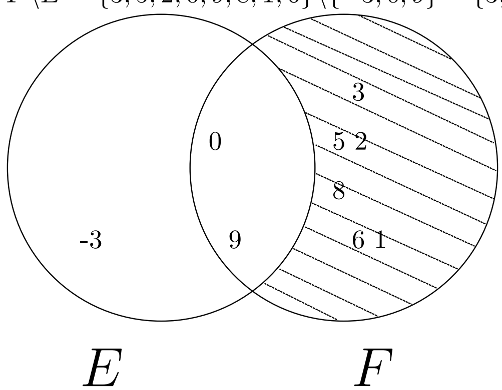
(2) $E \cup F = \{-3, 0, 9\} \cup \{3, 5, 2, 0, 9, 8, 1, 6\} = \{-3, 3, 5, 2, 0, 9, 8, 6, 1\}$



(3) $E \backslash F = \{-3, 0, 9\} \backslash \{3, 5, 2, 0, 9, 8, 1, 6\} = \{-3\}$



$(\mathbf{4})$ $F\backslash E = \{3,5,2,0,9,8,1,6\}\backslash\{-3,0,9\} = \{3,5,2,8,1,6\}$



3. $(\mathbf{1})$ $F = \{-1,0,4,8,6,1\}$

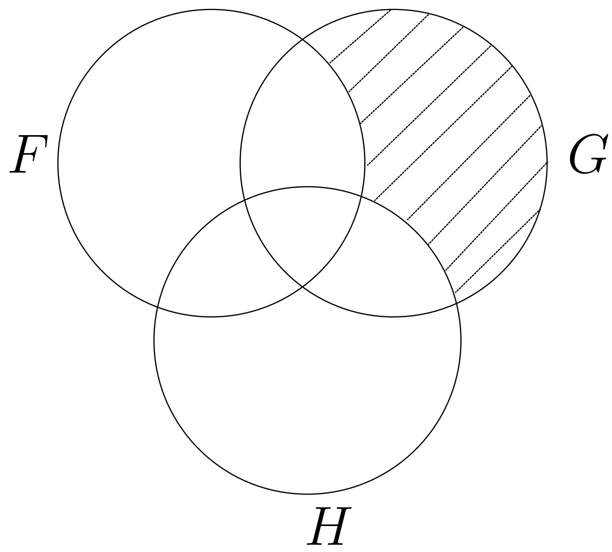
$(\mathbf{2})$ $F \cup H = \{-1,0,4,8,6,1\} \cup \{5,2,8\} = \{-1,5,2,0,4,8,6,1\}$

$(\mathbf{3})$ $G \cap F = \{3,-1,7,9,4,6\} \cap \{-1,0,4,8,6,1\} = \{-1,4,6\}$

$(\mathbf{4})$ $H\backslash F = \{5,2,8\}\backslash\{-1,0,4,8,6,1\} = \{5,2\}$

$(\mathbf{5})$

$$\begin{aligned} G\backslash (F\cup H) &= \{3,-1,7,9,4,6\}\backslash (\{-1,0,4,8,6,1\}\cup\{5,2,8\}) \\ &= \{3,-1,7,9,4,6\}\backslash\{-1,5,2,0,4,8,6,1\} \\ &= \{3,7,9\} \end{aligned}$$



(**6**)

$$
\begin{aligned}
(G \backslash H) \backslash F &= (\{3, -1, 7, 9, 4, 6\} \backslash \{5, 2, 8\}) \backslash \{-1, 0, 4, 8, 6, 1\} \\
&= \{3, -1, 7, 9, 4, 6\} \backslash \{-1, 0, 4, 8, 6, 1\} \\
&= \{3, 7, 9\}
\end{aligned}
$$

(**7**)

$$
\begin{aligned}
H \cup (F \cap G) &= \{5, 2, 8\} \cup (\{-1, 0, 4, 8, 6, 1\} \cap \{3, -1, 7, 9, 4, 6\}) \\
&= \{5, 2, 8\} \cup \{-1, 4, 6\} \\
&= \{5, -1, 2, 4, 8, 6\}
\end{aligned}
$$

(**8**) $\emptyset \cup H = \emptyset \cup \{5, 2, 8\} = \{5, 2, 8\}$

(**9**)

$$
\begin{aligned}
(F \backslash G) \cup (G \backslash H) &= (\{-1, 0, 4, 8, 1, 6\} \backslash \{3, -1, 7, 9, 4, 6\}) \cup (\{3, -1, 7, 9, 4, 6\} \backslash \{5, 2, 8\}) \\
&= \{0, 8, 1\} \cup \{3, -1, 7, 9, 4, 6\} \\
&= \{3, -1, 7, 0, 9, 4, 8, 1, 6\}
\end{aligned}
$$

4. (**1**) $Prob\,(s_1 \text{ is even}) = \dfrac{2}{5}$

(**2**) $Prob\,(s_1 = 8) = \dfrac{1}{5}$

(**3**) $Prob\,(s_1 \geq 8) = \dfrac{2}{5}$

(**4**) $Prob\,(s_1 \text{ is even and } s_1 \geq 8) = \dfrac{1}{5}$

(**5**) $Prob\,(s_1 \text{ is even or } s_1 \geq 8) = \dfrac{3}{5}$

(**6**) $Prob\,(s_1 \text{ is even given that } s_1 \geq 8) = \dfrac{1}{2}$

(**7**) $Prob\,(s_1 \text{ is even}) = \dfrac{2}{5}$, and $Prob\,(s_2 \text{ is even}) = \dfrac{2}{4} = \dfrac{1}{2}$.

Now s_1 and s_2 are chosen independently,

so $Prob\,(\text{both } s_1 \text{ and } s_2 \text{ are even }) = Prob\,(s_1 \text{ is even}) \times Prob\,(s_2 \text{ is even})$.

Hence $Prob\,(\text{both } s_1 \text{ and } s_2 \text{ are even }) = \dfrac{2}{5} \times \dfrac{1}{2} = \dfrac{1}{5}$

(**8**) By the principle of inclusion\exclusion,

$Prob\,(s_1 \text{ is even } \textbf{or } s_2 \text{ is even}) = Prob\,(s_1 \text{ is even}) + Prob\,(s_2 \text{ is even}) - Prob\,(\text{both } s_1 \text{ and } s_2 \text{ are even })$.

Hence $Prob\,(s_1 \text{ is even } \textbf{or } s_2 \text{ is even}) = \dfrac{2}{5} + \dfrac{1}{2} - \dfrac{1}{5} = \dfrac{7}{10}$

(**9**) Now s_1 and s_2 are chosen independently, so

$Prob\,(s_1 \text{ is even } \textbf{given that } s_2 \text{ is odd }) = Prob\,(s_1 \text{ is even})$.

Hence $Prob\,(s_1 \text{ is even } \textbf{given that } s_2 \text{ is odd }) = \dfrac{2}{5}$

- **STRAIGHT LINES AND SIMULTANEOUS EQUATIONS**

1. Let $(x_1, y_1) = (-6, 9)$ and $(x_2, y_2) = (-2, -10)$. Then $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, so
$d = \sqrt{(-6-(-2))^2 + (9-(-10))^2} = \sqrt{(-4)^2 + 19^2} = \sqrt{16+361} = \sqrt{377}$.
Hence $d = \sqrt{377}$

2. Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 1 + 10x &= -7y, \quad \text{so} \\ 7y &= -10x - 1 \\ y &= -\frac{10}{7}x - \frac{1}{7} \end{aligned}$$

Hence the gradient is $m = -\dfrac{10}{7}$ and the y-intercept is $c = -\dfrac{1}{7}$.

3. Rewrite the equation as $y = mx + c$:

$$\begin{aligned} 2 + 3x + 4y &= 8y + 7 - 7x, \text{ so} \\ 4y - 8y &= -7x - 3x + 7 - 2 \\ -4y &= -10x + 5 \\ y &= \frac{5}{2}x - \frac{5}{4} \end{aligned}$$

Hence the gradient is $m = \dfrac{5}{2}$ and the y-intercept is $c = -\dfrac{5}{4}$.

4. Thus the equation of the line is $y = 2x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (0, -9)$ into this equation to get the value for c. Hence $-9 = 2 \times 0 + c$, so $-9 = c$.
Hence the equation of the line is $y = 2x - 9$.

5. Let $(x_1, y_1) = (-7, 7)$ and $(x_2, y_2) = (9, 0)$. To find the equation of the line through (x_1, y_1) and (x_2, y_2) you must find the gradient m and the y-intercept c.

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 7}{9 - (-7)} = \dfrac{-7}{16}$. Hence $m = -\dfrac{7}{16}$.

Thus the equation of the line is $y = -\dfrac{7}{16}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) =$

$(-7, 7)$ into this equation to get the value for c.
Hence $7 = -\frac{7}{16} \times (-7) + c$, so $7 = \frac{49}{16} + c$. Hence $c = 7 - \frac{49}{16} = \frac{63}{16}$.
Hence the equation of the line is $y = -\frac{7}{16}x + \frac{63}{16}$.

6. To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} -15x &= 5y + 20, \text{ so} \\ -5y &= 15x + 20 \\ y &= -3x - 4 \end{aligned}$$

Hence, the gradient of the original line is $m = -3$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = -3x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-4, 10)$ into this equation to get the value for c.
$10 = -3 \times (-4) + c$, so $10 = 12 + c$. Hence $c = 10 - 12 = -2$.
Hence the equation of the line is $y = -3x - 2$.

7. To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 6y - 4 + 9x &= 5y - 10 + 13x, \text{ so} \\ 6y - 5y &= 13x - 9x - 10 + 4 \\ y &= 4x - 6 \end{aligned}$$

Hence, the gradient of the original line is $m = 4$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = 4x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (0, 8)$ into this equation to get the value for c.
$8 = 4 \times 0 + c$, so $8 = c$.
Hence the equation of the line is $y = 4x + 8$.

8. To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 6y &= 18x - 36, \text{ so} \\ y &= 3x - 6 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 3$.
The new line is perpendicular to the original line, so the new line has gradient $m = -\frac{1}{m_0}$. Hence $m = -\frac{1}{3}$.
Thus the equation of the line is $y = -\frac{1}{3}x + c$ and we can substitute the coordinates of the point $(x_1, y_1) = (-27, 19)$ into this equation to get the value of c:
$19 = -\frac{1}{3} \times (-27) + c$, so $19 = 9 + c$. Hence $c = 19 - 9 = 10$.
Hence the equation of the line is $y = -\frac{1}{3}x + 10$.

9. To determine whether the given line passes through the point $(x_1, y_1) = (-2, 11)$, we need to substitute the coordinates of the point into the equation of the line. Now,

$$\begin{aligned} 70x &= -7y - 63, \text{ so} \\ 70 \times (-2) &= -7 \times 11 - 63 \\ -140 &= -77 - 63 \\ -140 &= -140 \end{aligned}$$

The last statement is **true**, so our line **does** pass through the point $(-2, 11)$.

10. To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 21 &= 3y, \text{ so} \\ -3y &= -21 \\ y &= 7 \end{aligned}$$

Hence the gradient of the original line is $m_0 = 0$.
The original line is horizontal (its gradient is equal to 0), so the new line is vertical and has an equation of the form $x = c$. The point $(-2, -9)$ lies on the new line, so the equation of the new line is $x = -2$.

11. The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the line perpendicular to it will be horizontal with equation of the form $y = c$, where c is a constant.
The point $(-7, 0)$ lies on the new line, so the equation of the new line is $y = 0$.

12. The original line has an infinite gradient; it is vertical and parallel to the y-axis. Therefore the new line is vertical and has the form $x = c$, where c is a constant.
The point $(0, 9)$ lies on the new line, so the equation of the new line is $x = 0$.

13. To find the equation of the new line, we first need the gradient of the original line. Now,

$$\begin{aligned} 0 &= 70 + 10y, \text{ so} \\ -10y &= 70 \\ y &= -7 \end{aligned}$$

Hence, the gradient of the original line is $m = 0$.
The new line is parallel to the original line, so it has the same gradient as the original line. Thus the equation of the line is $y = c$ and we can substitute the coordinates of the point $(x_1, y_1) = (2, 8)$ into this equation to get the value for c.
$8 = c$.
Hence the equation of the line is $y = 8$.

14. First we number the equations for convenience:

$$\begin{aligned} -9y &= -4x - 540 \qquad (1) \\ 60x - 20 &= -10y \qquad (2) \end{aligned}$$

We solve these using substitution. Dividing both sides of equation (2) by -10 gives

$$-6x + 2 = y \qquad (3)$$

Substituting for y in equation (1),

$$-9 \times (-6x + 2) = -4x - 540 \qquad (4)$$

Now (4) is an equation only involving x which gives:

$$\begin{aligned} 54x - 18 &= -4x - 540 \\ 58x &= -522 \\ x &= -9 \end{aligned}$$

Next we substitute the value for x into equation (3) to obtain the value for y, giving

$$y = -6 \times (-9) + 2 = 56$$

Hence the simultaneous solution to equations (1) and (2) is $(-9, 56)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1) \quad -9 \times 56 &= -4 \times (-9) - 540 \\ -504 &= 36 - 540 \\ -504 &= -504 \end{aligned}$$

$$\begin{aligned} (2) \quad 60 \times (-9) - 20 &= -10 \times 56 \\ -540 - 20 &= -560 \\ -560 &= -560 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

15. First we number the equations for convenience:

$$\begin{aligned} 2y - 6x &= -6 \qquad (1)\\ 6y - 18x &= -22 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by -3 , giving

$$\begin{aligned} -6y + 18x &= 18 \qquad (3)\\ 6y - 18x &= -22 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-6y + 6y + 18x - 18x = 18 - 22 \qquad (5)$$

Simplifying equation (5) gives

$$0 = -4 \qquad (6)$$

Statement (6) is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.

16. Let $z = \tan x$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} 3y - 5z &= -11 \qquad (1)\\ 3y - 10z &= -16 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (2) by -1 , giving

$$\begin{aligned} 3y - 5z &= -11 \qquad (3)\\ -3y + 10z &= 16 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-3y + 3y + 10z - 5z = 16 - 11 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} 5z &= 5 \qquad (6)\\ z &= 1 \qquad (7) \end{aligned}$$

Next we substitute the value for z into equation (1) to obtain the value for y, giving

$$\begin{aligned} 3y - 5 \times 1 &= -11\\ 3y &= -6 \qquad \text{so}\\ y &= -2 \end{aligned}$$

Now we can find the value of x: $\quad \tan x = 1, \quad$ so $x = \dfrac{\pi}{4}; \dfrac{5\pi}{4}$

Hence the simultaneous solution to equations (1) and (2) is $x = \dfrac{\pi}{4}; \dfrac{5\pi}{4}; \quad y = -2.$

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ \ 3 \times (-2) - 5 \times \tan\frac{\pi}{4} &= -11\\ 3 \times (-2) - 5 \times 1 &= -11\\ -6 - 5 &= -11\\ -11 &= -11 \end{aligned}$$

$$\begin{aligned} (2)\ \ 3 \times (-2) - 10 \times \tan\frac{\pi}{4} &= -16\\ 3 \times (-2) - 10 \times 1 &= -16\\ -6 - 10 &= -16\\ -16 &= -16 \end{aligned}$$

We have checked one value of x, you do the other!)

17. We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$-8y - 7x = 12 \qquad (1)$$
$$-2y + 5x = 30 \qquad (2)$$

It's probably easier to solve these using elimination. Multiply equation (2) by -4 , giving

$$-8y - 7x = 12 \qquad (3)$$
$$8y - 20x = -120 \qquad (4)$$

We add both sides of equations (3) and (4) , giving

$$8y - 8y - 20x - 7x = -120 + 12 \qquad (5)$$

Simplifying equation (5) gives

$$-27x = -108 \qquad (6)$$
$$x = 4 \qquad (7)$$

Next we substitute the value for x into equation (1) to obtain the value for y, giving

$$-8y - 7 \times 4 = 12$$
$$-8y = 40 \qquad \text{so}$$
$$y = -5$$

Hence the simultaneous solution to equations (1) and (2) is $(4, -5)$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$(1) \quad -8 \times (-5) - 7 \times 4 = 12 \qquad\qquad (2) \quad -2 \times (-5) + 5 \times 4 = 30$$
$$40 - 28 = 12 \qquad\qquad 10 + 20 = 30$$
$$12 = 12 \qquad\qquad 30 = 30$$

Both equations turned into true statements, as required. Hence the answer is correct.)

18. We need to find a solution for two simultaneous linear equations.
First we number the equations for convenience:

$$100y + 10x + 91 = 0 \qquad (1)$$
$$-6x - 54 - 60y = 0 \qquad (2)$$

We solve these using substitution. Rearranging equation (2) with x on the left-hand side gives

$$-6x = 60y + 54 \qquad (3)$$

Dividing both sides of (3) by -6, gives

$$x = -10y - 9 \qquad (4)$$

Substituting for x in equation (1),

$$100y + 10 \times (-10y - 9) + 91 = 0 \qquad (5)$$

Now (5) is an equation only involving y which gives:

$$100y - 100y - 90 + 91 = 0$$
$$1 = 0$$

This statement is **never true**, so there is no solution to our simultaneous equations. The lines are parallel.

- **FUNCTIONS, QUADRATICS AND TRIGONOMETRIC FUNCTIONS**

1. **(1)** $y = 2 \times |-7x|$, so $y = 2 \times |7x|$, which is a graph of absolute value. Hence the matching graph is Graph N.

(2) $y = e^{-7x}$, which is a graph of exponential decay. Hence the matching graph is Graph L.

(3) $x = -15y - 12x$, so $15y = -13x$. Hence this is a straight line, with negative gradient and passing through the origin. Hence the matching graph is Graph I.

(4) $-9 = -y + 9x$, so $y = 9x + 9$. Hence this is a straight line, with positive gradient and positive y-intercept. Hence the matching graph is Graph G.

(5) $y - 10x - 8 = 3y - 13x - 8$, so $2y = 3x$. Hence this is a straight line, with positive gradient and passing through the origin. Hence the matching graph is Graph F.

(6) $7y + 14x^2 = 12y + 15x^2$, so $5y = -x^2$. This equation includes an x^2 term with a negative coefficient, so the graph is a parabola which turns downwards. Also, the y-intercept is 0. Hence the matching graph is Graph S.

(7) $y = e^{2x}$, which is a graph of exponential growth. Hence the matching graph is Graph K.

(8) $0 = 13y + 10x + 12$, so $13y = -10x - 12$. Hence this is a straight line, with negative gradient and negative y-intercept. Hence the matching graph is Graph J.

2. $f(x) = \left(\left|\frac{8}{x}\right|\right)^2$

When determining the domain of this function, we need to keep in mind the following:

- we can square any number;
- we can find the absolute value of any number;
- denominator of a fraction cannot be 0, so $x \neq 0$.

Hence, the domain of this function is $(-\infty, 0) \cup (0, \infty)$, i.e. $x \neq 0$.

3. $f(w) = \frac{-6}{|w| + 2}$

When determining the domain of this function, we need to keep in mind the following:

- denominator of a fraction cannot be 0, so $|w| + 2 \neq 0$;
- so $|w| \neq -2$;
- we can find the absolute value of any number. It will always be positive or 0.

Hence, the domain of this function is $(-\infty, \infty)$, i.e. any value of w can be substituted into f .

4. $f(x) = \frac{7}{(-4x)^2}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- there are no square roots or absolute value signs;
- squaring always gives a positive or 0, so $(-4x)^2 \geq 0$;
- fraction can be 0 only if numerator is 0, so $\frac{7}{(-4x)^2} > 0$.

Hence, the range of this function is $(0, \infty)$.

5. $f(x) = \frac{1}{12 - 6x}$

When evaluating the range, we need to keep in mind the following (starting with variable x):

- there are no squares, square roots or absolute value signs ;

– fraction can be 0 only if its numerator is 0 (which is not the case), denominator cannot be 0 .

Hence, the range of this function is $(-\infty, 0) \cup (0, \infty)$.

6. $f(w) = \left|\dfrac{-9}{-8+w}\right|$

When determining the domain of this function, we need to keep in mind the following:

– we can find the absolute value of any number;
– denominator of a fraction cannot be 0, so $-8+w \neq 0$;
– so $w \neq 8$.

Hence, the domain of this function is $(-\infty, 8) \cup (8, \infty)$, i.e. $w \neq 8$.

When evaluating the range, we need to keep in mind the following (starting with variable w):

– negative numerator usually reverse the inequality, and also this fraction can't be 0, so $\dfrac{-9}{-8+w} \neq 0$;
– absolute value is always positive or 0, so $\left|\dfrac{-9}{-8+w}\right| > 0$.

Hence, the range of this function is $(0, \infty)$.

7. $3y^2 - 9y - 12 = 0$, so we use $a = 3, b = -9, c = -12$ in the quadratic formula. Hence

$$\begin{aligned} y &= \frac{9 \pm \sqrt{(-9)^2 - 4 \times 3 \times (-12)}}{2 \times 3} \\ &= \frac{9 \pm \sqrt{81 - (-144)}}{6} \\ &= \frac{9 \pm \sqrt{225}}{6} \\ &= \frac{9+15}{6} \quad or \quad \frac{9-15}{6} \\ &= \frac{24}{6} \quad or \quad \frac{-6}{6} \\ &= 4 \quad or \quad -1 \end{aligned}$$

8. To solve each of these, remember that if $a \times b = 0$, then either $a = 0$ or $b = 0$; and also that $0^n = 0$ for any natural number n. Then:

(**1**) $-6z(-8+9z) = 0$, so

$$\begin{aligned} -6z &= 0 \qquad & or \qquad & -8+9z = 0 \\ z &= 0 & & 9z = 8 \\ & & & z = \frac{8}{9} \end{aligned}$$

(**2**) $(-4z+10)(4+7z) = 0$, so

$$\begin{aligned} -4z+10 &= 0 \qquad & or \qquad & 4+7z = 0 \\ -4z &= -10 & & 7z = -4 \\ z &= \frac{-10}{-4} & & z = -\frac{4}{7} \\ z &= \frac{5}{2} & & \end{aligned}$$

(**3**) $(4x+5)(-9+x) = 0$, so

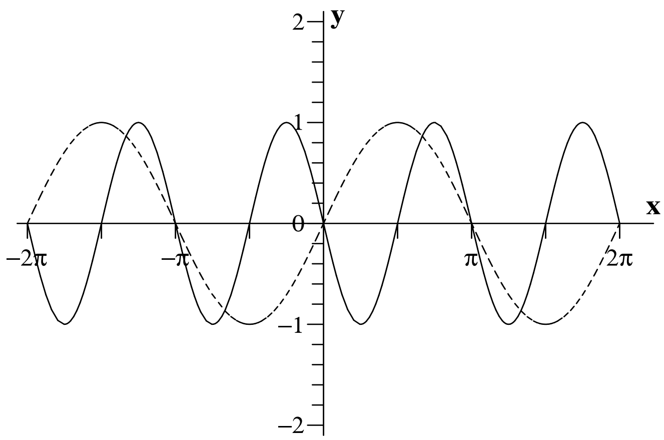
$$\begin{aligned} 4x+5 &= 0 \qquad & or \qquad & -9+x = 0 \\ 4x &= -5 & & x = 9 \\ x &= -\frac{5}{4} & & \end{aligned}$$

(**4**) $(3y+8)^6 = 0$, so $3y+8=0$, so $3y=-8$, so $y = -\dfrac{8}{3}$

9. $(1-2z)^8 = 0$, so $1-2z=0$, so $-2z=-1$, so $z=\dfrac{1}{2}$

10. $f(x) = 5x^2+4x-8$, so
$f(3) = 5\times 3^2 + 4\times 3 - 8 = 45+12-8 = 49$

11. The graph of $y = \sin x$ is dashed; the graph of $y_1 = -\sin(2x)$ is solid.



12. Given an angle a in radians, to convert a to degrees you multiply by 180 and divide by π. Hence the converted angles are:

$$-144^\circ \quad -225^\circ \quad 63^\circ \quad -345^\circ \quad -27^\circ \quad 345^\circ \quad -189^\circ \quad -660^\circ$$

13. Given an angle a in degrees, to convert a to radians you divide by 180 and multiply by π. Hence the converted angles are:

$$-\frac{10\pi}{9} \quad -\frac{21\pi}{10} \quad \pi \quad \frac{3\pi}{4} \quad -\frac{6\pi}{5} \quad -\pi \quad -\frac{5\pi}{3} \quad -6\pi$$

• LOGARITHMS AND EXPONENTIALS

1. (**1**) $\log_{12} 12^{12} = 12$

(**2**) $3 = 3^1$, so $\log_3 3 = 1$

(**3**) $\dfrac{1}{125} = 5^{-3}$, so $\log_5 \dfrac{1}{125} = \log_5 5^{-3} = -3$. Hence the answer is -3.

(**4**) $10 = 10^1$, so $\log_{10} 10 = 1$

(**5**) $\dfrac{1}{1000000} = 10^{-6}$, so $\log_{10} \dfrac{1}{1000000} = -6$

(**6**) $1 = e^0$, so $\ln 1 = 0$

(**7**) $\dfrac{1}{e^3} = e^{-3}$, so $\ln \dfrac{1}{e^3} = \ln e^{-3} = -3$. Hence the answer is -3.

(**8**) $4 = 16^{\frac{1}{2}}$, so $\log_{16} 4 = \dfrac{1}{2}$

2. Let P be the final population in millions. Then

$$\begin{aligned} P &= 800e^{0.02\times 6} \\ &= 800e^{0.12} \\ &\approx 902.00 \end{aligned}$$

Hence the final population is approximately 902.00 million bacteria.

3. Let A be the final amount of the material remaining. Then

$$\begin{aligned} A &= 100e^{-0.10\times 3} \\ &= 100e^{-0.3} \\ &\approx 74.08 \end{aligned}$$

Hence the amount of material remaining after 3 thousand years is approximately 74.08 units.

4. Let P be the amount invested, r be the interest rate per time period, n be the number of time periods and F be the final value. In each case, $P = 400$. Then:

(1) Interest compounds annually, so we use the rate and number of time periods given in the question.
Hence $r = 7.0\% = 0.07$ and $n = 1$, so $F = 400 \times (1 + 0.07)^1 = 400 \times 1.07^1 = 428.00$.
The final balance is \$428.00.

(2) Interest compounds twice a year, so we need to halve the rate and double the number of time periods given in the question.
Hence $r = 3.5\% = 0.035$ and $n = 2$, so $F = 400 \times (1 + 0.035)^2 = 400 \times 1.035^2 \approx 428.49$.
The final balance is \$428.49.

(3) Interest compounds 4 times a year, so we need to divide the given rate by 4 and multiply the given number of years by 4.
Hence $r = 1.8\% = 0.0175$ and $n = 4$, so $F = 400 \times (1 + 0.0175)^4 = 400 \times 1.0175^4 \approx 428.74$.
The final balance is \$428.74.

(4) Interest compounds 12 times a year, so we need to divide the given rate by 12 and multiply the given number of years by 12.
Hence $r = 0.6\% = 0.0058$ and $n = 12$, so $F = 400 \times (1 + 0.0058)^{12} = 400 \times 1.0058^{12} \approx 428.92$.
The final balance is \$428.92.

(5) Interest compounds continuously, so $F = 400e^{0.07\times 1} = 400e^{0.07} \approx 429.00$.
The final balance is \$429.00.

5. Let growth rate be k. Then

$$\begin{aligned} 2 &= e^{k\times h}, \text{ so} \\ \ln 2 &= kh, \text{ so} \\ k &= \frac{\ln 2}{h} \\ &= \frac{\ln 2}{38.5} \\ &\approx 0.018 \end{aligned}$$

Hence the growth rate is approximately 0.018 per hour.

6. Let doubling time be t. Then

$$\begin{aligned} 2 &= e^{t\times k}, \text{ so} \\ \ln 2 &= tk, \text{ so} \\ t &= \frac{\ln 2}{k} \\ &= \frac{\ln 2}{0.06} \\ &\approx 11.5525 \end{aligned}$$

Hence the doubling time is approximately 11.5525 hour(s).

7. Let B be the amount of the bill, I be the amount he needs to invest, r be the interest rate and t be the number of years. Then $B = Ie^{rt}$ so $I = \frac{B}{e^{rt}}$, so $I = Be^{-rt}$. Then

$$\begin{aligned} I &= 800e^{-0.08\times 7} \\ &= 800e^{-0.56} \\ &\approx 456.97 \end{aligned}$$

Hence he needs to invest approximately \$456.97 .

8. Let F be the final account balance. Then

$$\begin{aligned} F &= 1000e^{0.06\times 7} \\ &= 1000e^{0.42} \\ &\approx 1521.96 \end{aligned}$$

Hence the final account balance is approximately \$1521.96 .

9. Let F be the final amount he needs, I be the amount he has to invest, r be the interest rate and t be the number of years. Then $F = Ie^{rt}$ so $e^{rt} = \frac{F}{I}$, so $rt = \ln \frac{F}{I}$, and $t = \left(\ln \frac{F}{I}\right) \div r$. Then

$$\begin{aligned} t &= \left(\ln \frac{1200}{600}\right) \div 0.08 \\ &= (\ln 2.00) \div 0.08 \\ &\approx 0.69 \div 0.08 \\ &\approx 8.66 \end{aligned}$$

Hence he needs to invest \$600 for approximately 8.66 years. Therefore Damien can marry Celeste when he is about 19 years old.

10. Let B be the price of the shoes, I be the amount Peter needs to invest, n be the number of compounding periods before the Congress, r be the interest compounding quarterly . Then

$r = 3 \times \frac{7.0}{12} = 1.75$ percent $= 0.0175$, and

$n = 9 \div 3 = 3$

$B = I\left(1 + r\right)^n$, so $I = \frac{B}{(1+r)^n}$. Therefore

$$\begin{aligned} I &= \frac{400}{(1+0.0175)^3} \\ &= \frac{400}{1.0534} \\ &\approx 379.71 \end{aligned}$$

Hence he needs to invest approximately \$379.71 .

- **DERIVATIVES AND INTEGRATION**

1. $y = 8$, so

$$y' = 0$$

2. $y = 3x^2 + 3$, so

$$\begin{aligned} y' &= 2 \times 3x^{2-1} \\ &= 6x \end{aligned}$$

3. $y = -7x^6$, so

$$y' = 6 \times \left(-7x^{6-1}\right)$$
$$= -42x^5$$

4. $y = -\cos x$, so

$$y' = -1 \times (-\sin x)$$
$$= \sin x$$

5. $y = 6e^x$, so

$$y' = 6e^x$$

6. $y = -\frac{8}{x} + 4\sin x - 7\sqrt{x} + 2\sin x$, so $y = -8x^{-1} + 4\sin x - 7x^{\frac{1}{2}} + 2\sin x$, so

$$y' = -1 \times \left(-8x^{-1-1}\right) + 4\cos x - \frac{1}{2} \times 7 \times x^{\frac{1}{2}-1} + 2\cos x$$
$$= 8x^{-2} + 4\cos x - \frac{7}{2}x^{-\frac{1}{2}} + 2\cos x$$
$$= \frac{8}{x^2} + 4\cos x - \frac{7}{2\sqrt{x}} + 2\cos x$$

Hence $y' = \frac{8}{x^2} + 4\cos x - \frac{7}{2\sqrt{x}} + 2\cos x$.

7. Q1 $f'(x) = -3x^2 - 6x + 9$

Q2 $f'(x) = 0$, so from Q1 , $-3x^2 - 6x + 9 = 0$, so we use $a = -3, b = -6, c = 9$ in the quadratic formula. Hence

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times (-3) \times 9}}{2 \times (-3)}$$
$$= \frac{6 \pm \sqrt{36 - (-108)}}{-6}$$
$$= \frac{6 \pm \sqrt{144}}{-6}$$
$$= \frac{6 + 12}{-6} \quad or \quad \frac{6 - 12}{-6}$$
$$= \frac{18}{-6} \quad or \quad \frac{-6}{-6}$$
$$= -3 \quad or \quad 1$$

Q3 $f''(x) = -6x - 6$

Q4 $f'(5) = -3 \times 5^2 - 6 \times 5 + 9 = -96$

8. Let $u = 10 + 9x^{-2}$, so $y = \frac{1}{u^7} = u^{-7}$.

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$\frac{dy}{du} = -7 \times u^{-7-1} = -7u^{-8}$

$\frac{du}{dx} = 9 \times (-2) \times x^{-2-1} = -18x^{-3}$

So, $\frac{dy}{dx} = -7u^{-8} \times \left(-18x^{-3}\right) = -7\left(10 + 9x^{-2}\right)^{-8} \times \left(-18x^{-3}\right) = 126x^{-3}\left(10 + 9x^{-2}\right)^{-8} = \frac{126}{x^3\left(10 + 9x^{-2}\right)^8}$.

Hence $\frac{dy}{dx} = \frac{126}{x^3\left(10 + 9x^{-2}\right)^8}$.

9. Let $u = 9r - 6$, then $u' = 9$.
Let $v = 4r + 10$, then $v' = 4$.
Quotient rule: $y' = \dfrac{u'v - uv'}{v^2}$, so

$$y' = \frac{9 \times (4r+10) - (9r-6) \times 4}{(4r+10)^2} = \frac{36r + 90 - 36r + 24}{(4r+10)^2} = \frac{114}{(4r+10)^2}.$$

Hence $y' = \dfrac{114}{(4r+10)^2}$.

10. Let $u = -9x - 8 - 7x^2$, then $u' = -9 - 14x$.
Let $v = -1 + 6x^2 - 9x$, then $v' = 12x - 9$.
Quotient rule: $y' = \dfrac{u'v - uv'}{v^2}$, so

$$\begin{aligned} y' &= \frac{(-9-14x) \times \left(-1+6x^2-9x\right) - \left(-9x-8-7x^2\right) \times (12x-9)}{(-1+6x^2-9x)^2} \\ &= \frac{9 - 54x^2 + 81x + 14x - 84x^3 + 126x^2 + 108x^2 - 81x + 96x - 72 + 84x^3 - 63x^2}{(-1+6x^2-9x)^2} \end{aligned}$$

Hence $y' = \dfrac{117x^2 + 110x - 63}{(-1+6x^2-9x)^2}$.

11. Let $u = -6t^2 + 9$, then $u' = -12t$.
Let $v = 3t^2 + 10t$, then $v' = 6t + 10$.
Quotient rule: $y' = \dfrac{u'v - uv'}{v^2}$, so

$$\begin{aligned} y' &= \frac{-12t \times \left(3t^2+10t\right) - \left(-6t^2+9\right) \times (6t+10)}{(3t^2+10t)^2} \\ &= \frac{-36t^3 - 120t^2 - \left(-36t^3 - 60t^2 + 54t + 90\right)}{(3t^2+10t)^2} \\ &= \frac{-36t^3 - 120t^2 + 36t^3 + 60t^2 - 54t - 90}{(3t^2+10t)^2} \end{aligned}$$

Hence $y' = \dfrac{-60t^2 - 54t - 90}{(3t^2+10t)^2}$.

12. Let $u = 8t - 3$, then $u' = 8$.
Let $v = 6t^3 - 3$, then $v' = 18t^2$.
Product rule: $y' = u'v + uv'$.
Substitute u, u', v and v' into the product rule:

$$\begin{aligned} y' &= 8 \times \left(6t^3 - 3\right) + (8t-3) \times 18t^2 \\ &= 48t^3 - 24 + 144t^3 - 54t^2 \end{aligned}$$

Hence $y' = 192t^3 - 54t^2 - 24$.

13. $\displaystyle\int (10x^4 - 7x + 5)\, dx = 2x^5 - \frac{7}{2}x^2 + 5x + C.$

14.

$$\int_{-3}^{0}\left(9x^2-8x+2\right)dx = \Big[3x^3-4x^2+2x\Big]_{-3}^{0}$$
$$= \left(3\times 0^3-4\times 0^2+2\times 0\right)-\left(3\times(-3)^3-4\times(-3)^2+2\times(-3)\right)$$
$$= 0-(-81-36-6)$$
$$= 0-(-123)$$
$$= 123$$

15.

$$\int_{3\pi/2}^{5\pi/2}\left(-3\sin\left(-3x\right)\right)dx$$
$$= \Big[-\cos\left(-3x\right)\Big]_{3\pi/2}^{5\pi/2}$$
$$= \Big(-\cos\left(-3\left(5\pi/2\right)\right)\Big)-\Big(-\cos\left(-3\left(3\pi/2\right)\right)\Big)$$
$$= \Big(-\cos\left(-15\pi/2\right)\Big)-\Big(-\cos\left(-9\pi/2\right)\Big)$$
$$= \Big(-(0)\Big)-\Big(-(0)\Big)$$
$$= (0)-(0)$$
$$= 0$$

- **MATRICES**

1. $\begin{pmatrix} 0 & 6 & -3 \\ 5 & 6 & -4 \end{pmatrix} + \begin{pmatrix} -6 & -6 & -3 \\ 6 & -1 & -5 \end{pmatrix} = \begin{pmatrix} -6 & 0 & -6 \\ 11 & 5 & -9 \end{pmatrix}$

2. $\begin{pmatrix} 5 & -2 \\ -2 & 3 \\ -2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -3 & 0 \\ -7 & 1 \end{pmatrix}$

3. $\begin{pmatrix} -2 & 0 & -5 \\ -4 & -5 & -2 \end{pmatrix} \times \begin{pmatrix} -5 & -2 \\ 2 & 1 \\ -6 & -5 \end{pmatrix}$

$$= \begin{pmatrix} -2\times(-5)+0\times 2-5\times(-6) & -2\times(-2)+0\times 1-5\times(-5) \\ -4\times(-5)-5\times 2-2\times(-6) & -4\times(-2)-5\times 1-2\times(-5) \end{pmatrix}$$
$$= \begin{pmatrix} 40 & 29 \\ 22 & 13 \end{pmatrix}$$

4.

$$\begin{pmatrix} -3 & 1 \\ -6 & -1 \end{pmatrix}^{-1} = \frac{1}{-3\times(-1)-1\times(-6)}\begin{pmatrix} -1 & -1 \\ 6 & -3 \end{pmatrix}$$
$$= \frac{1}{9}\begin{pmatrix} -1 & -1 \\ 6 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} -1/9 & -1/9 \\ 2/3 & -1/3 \end{pmatrix}$$

5. $-6\times\begin{pmatrix} 3 & 3 & -6 \end{pmatrix} = \begin{pmatrix} -6\times 3 & -6\times 3 & -6\times(-6) \end{pmatrix} = \begin{pmatrix} -18 & -18 & 36 \end{pmatrix}$

6.

$$0\begin{pmatrix}6\\2\\1\end{pmatrix}+\begin{pmatrix}2\\-1\\-3\end{pmatrix}+3\begin{pmatrix}3\\2\\0\end{pmatrix}$$
$$=\begin{pmatrix}0\\0\\0\end{pmatrix}+\begin{pmatrix}2\\-1\\-3\end{pmatrix}+\begin{pmatrix}9\\6\\0\end{pmatrix}$$
$$=\begin{pmatrix}2\\-1\\-3\end{pmatrix}+\begin{pmatrix}9\\6\\0\end{pmatrix}$$
$$=\begin{pmatrix}11\\5\\-3\end{pmatrix}$$

7. (a)

$$\begin{pmatrix}-3&-6&-4\\2&3&3\\3&-2&6\end{pmatrix}\begin{pmatrix}24&44&-6\\-3&-6&1\\-13&-24&3\end{pmatrix}$$
$$=\begin{pmatrix}-3\times 24-6\times(-3)-4\times(-13) & -3\times 44-6\times(-6)-4\times(-24) & -3\times(-6)-6\times 1-4\times 3\\ 2\times 24+3\times(-3)+3\times(-13) & 2\times 44+3\times(-6)+3\times(-24) & 2\times(-6)+3\times 1+3\times 3\\ 3\times 24-2\times(-3)+6\times(-13) & 3\times 44-2\times(-6)+6\times(-24) & 3\times(-6)-2\times 1+6\times 3\end{pmatrix}$$
$$=\ -2\mathbf{I}$$

(b) The set of simultaneous equations can be written in matrix form as

$$\begin{pmatrix}-3&-6&-4\\2&3&3\\3&-2&6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\1\\-2\end{pmatrix}$$

We know that

$$\begin{pmatrix}-3&-6&-4\\2&3&3\\3&-2&6\end{pmatrix}^{-1}=\frac{1}{-2}\begin{pmatrix}24&44&-6\\-3&-6&1\\-13&-24&3\end{pmatrix}$$

So,

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{-2}\begin{pmatrix}24&44&-6\\-3&-6&1\\-13&-24&3\end{pmatrix}\begin{pmatrix}4\\1\\-2\end{pmatrix}$$
$$=\frac{1}{-2}\begin{pmatrix}24\times 4+44\times 1-6\times(-2)\\-3\times 4-6\times 1+1\times(-2)\\-13\times 4-24\times 1+3\times(-2)\end{pmatrix}$$
$$=\frac{1}{-2}\begin{pmatrix}152\\-20\\-82\end{pmatrix}$$

So the solution to the simultaneous equations is $x=76$, $y=-10$ and $z=-41$.

- **BASIC PROGRAMMING WITH PYTHON**

1. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

(1)
```
>>> 4*(6-2)**(2*4)
262144
```
(2)
```
>>> 4-(4+2)+2-2
-2
```
(3)
```
>>> (4-5*5*2)*3
-138
```
(4)
```
>>> (6**(3+6))**(6-5)
10077696
```
(5)
```
>>> 6+6**2*4*5
726
```
(6)
```
>>> 3**4+3*5**3
456
```
(7)
```
>>> 6**(3+1)-5*5
1271
```
(8)
```
>>> 2*(1-(6+4*2))
-26
```
(9)
```
>>> (4-5*3)*3+1
-32
```
(10)
```
>>> (2-4)**(4*1**4)
16
```

2. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> sqrt((-4-0)**2+(-4-3)**2)
8.06225774829855
```

Thus, the distance between $(0, 3)$ and $(-4, -4)$ is 8.06225774829855.

3. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> sin(2)**2+cos(2)**2
1.0
```

We can see that the answer is indeed equal to 1, and so the expression is numerically verified.

4. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> sin(4*pi/180)
0.0697564737441253
>>> cos(4*pi/180)
0.9975640502598242
>>> tan(4*pi/180)
0.06992681194351041
```

5. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> log10(10**12)
12.0
>>> 10**log10(12)
12.0
```

We thus obtain a result of 12.0 for both, and so both expressions are numerically verified.

6. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> exp(-3)
0.049787068367863944
```

7. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> A=array([[0, 1], [13, 12]])
>>> B=inverse(A)
>>> B
array([[-0.9230769230769231, 0.07692307692307693],
       [1.0, -0.0]])
>>> dot(A,B)
array([[1.0, 0.0],
       [0.0, 1.0]])
>>> dot(B,A)
array([[1.0, 0.0],
       [0.0, 1.0]])
```

We can see that AB and BA are both approximately equal to the identity matrix.

8. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> A=array([[10, 3], [-14, 5]])
>>> b=array([[2], [16]])
>>> Ainv=inverse(A)
>>> dot(Ainv,b)
array([[-0.41304347826086957],
       [2.0434782608695654]])
```

We thus have that:

$$\begin{aligned} x &= -0.4130434782608696 \\ y &= 2.043478260869565 \end{aligned}$$

9. The full Python inputs and results are shown below. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

```
>>> from pylab import *
>>> for x in arange(-1, -21, -1):
        x**3

-1
-8
-27
-64
```

```
-125
-216
-343
-512
-729
-1000
-1331
-1728
-2197
-2744
-3375
-4096
-4913
-5832
-6859
-8000
```

- **SCIENCE**

1. The initial population vector P_0 and the Leslie matrix L are:

$$P_0 = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 3 & 8 \\ 0.4 & 0 \end{pmatrix}.$$

Then to find the population at time step $t+1$ we calculate $P_{t+1} = L \times P_t$, as follows:
At time $t = 1$ the population P_1 is given by:

$$\begin{pmatrix} 3 & 8 \\ 0.4 & 0 \end{pmatrix} \times \begin{pmatrix} 10 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \times 10 + 8 \times 5 \\ 0.4 \times 10 + 0 \times 5 \end{pmatrix} = \begin{pmatrix} 70 \\ 4 \end{pmatrix}$$

At time $t = 2$ the population P_2 is given by:

$$\begin{pmatrix} 3 & 8 \\ 0.4 & 0 \end{pmatrix} \times \begin{pmatrix} 70 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \times 70 + 8 \times 4 \\ 0.4 \times 70 + 0 \times 4 \end{pmatrix} = \begin{pmatrix} 242 \\ 28 \end{pmatrix}$$

At time $t = 3$ the population P_3 is given by:

$$\begin{pmatrix} 3 & 8 \\ 0.4 & 0 \end{pmatrix} \times \begin{pmatrix} 242 \\ 28 \end{pmatrix} = \begin{pmatrix} 3 \times 242 + 8 \times 28 \\ 0.4 \times 242 + 0 \times 28 \end{pmatrix} = \begin{pmatrix} 950 \\ 96.8 \end{pmatrix}$$

2. The chemical equation is: $3O_2(g) \rightarrow 2O_3(g)$

$$\begin{aligned} \text{The number of moles } O_3 \text{ produced} &= 6.90\text{mol } O_2 \times \frac{2\text{mol } O_3}{3\text{mol } O_2} \\ &= 4.60\text{mol } O_3 \end{aligned}$$

$$\begin{aligned} \text{Since } V/n \text{ is constant, } \frac{V_1}{n_1} &= \frac{V_2}{n_2} \\ \therefore V_2 &= \frac{n_2}{n_1} \times V_1 \\ &= \frac{4.60\text{mol}}{6.90\text{mol}} \times 11.2\text{L} \\ &= 7.5\text{L} \end{aligned}$$

3. Since PV=constant,

$$\begin{aligned} P_1V_1 &= P_2V_2, \quad \text{so} \\ P_2 &= \frac{P_1V_1}{V_2} \\ &= 104.33 \text{ kPa} \times \frac{504 \text{ mL}}{1128 \text{ mL}} \\ &= 46.62 \text{ kPa} \end{aligned}$$

or to convert to mmHg:

$$\begin{aligned} P_2 &= \frac{46.62 \text{ kPa}}{101.325\text{kPa}} \times 760 \text{ mmHg} \\ &= 350 \text{ mmHg} \end{aligned}$$

4. The number of moles of the gas remains constant, so $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\therefore\ P_2 = \frac{P_1V_1}{T_1} \times \frac{T_2}{V_2}$

Here,

$V_1 = 14.90 \times 10^{-3}\text{m}^3$ $\qquad$ $V_2 = 13.96 \times 10^{-3}\text{m}^3$

$T_1 = 42°\text{C} + 273 = 315\text{K}$ $\qquad$ $T_2 = -12°\text{C} + 273 = 261\text{K}$

$P_1 = 50.2\text{kPa}$ $\qquad$ $P_2 =?$

So $P_2 = \frac{50.2\text{kPa} \cdot 14.90 \times 10^{-3}\text{m}^3 \cdot 261\text{K}}{315\text{K} \cdot 13.96 \times 10^{-3}\text{m}^3} = 44.4\text{kPa}$

5. **(1)** Let us denote the mass and volume of the block as b and V respectively, the mass of Ben as m, the density of water as ρ_w and the density of the block as ρ_b. The mass of water w displaced by the block is then:

$$\begin{aligned} w &= \rho_w \times V \\ &= \rho_w \times \frac{b}{\rho_b} \\ &= \frac{\rho_w}{\rho_b} \times b \end{aligned}$$

The minimum amount of foam needed will be that which keeps the system in equilibrium (i.e. any more and the block will rise, any less and it will sink). Thus, the mass of water displaced must equal Ben's mass *as well as the mass of the block!*.

$$\begin{aligned} & & w &= m + b \\ &\implies & \frac{\rho_w}{\rho_b} \times b &= m + b \\ &\implies & \frac{\rho_w}{\rho_b} \times b - b &= m \\ &\implies & \left(\frac{\rho_w}{\rho_b} - 1\right) \times b &= m \\ &\implies & b &= \frac{m}{\frac{\rho_w}{\rho_b} - 1} \end{aligned}$$

Substituting in the values and rounding off to two decimal places, we have:

$$\begin{aligned} b &= \frac{100}{\frac{1000.0}{18} - 1} \\ &= 1.83\text{kg} \end{aligned}$$

Thus, Ben will need at least 1.83kg of foam to support his weight.

(**2**) Using the same symbols as above, let us further denote the percentage of the block that is *underwater* by p. That is:

$$p = 100\% - 40\% = 60\% = 0.6$$

The mass of water that is displaced is now thus dependent upon how much of the block is underwater, V', where:

$$V' = p \times V$$

Thus, the mass of water w' displaced with this reduced volume will be:

$$\begin{aligned} w' &= \rho_w \times V' \\ &= \rho_w \times (p \times V) \\ &= p\rho_w \times \frac{b}{\rho_b} \\ &= \frac{p\rho_w}{\rho_b} \times b \end{aligned}$$

Again, we want the block to be in equilibrium, so:

$$\begin{aligned} & w' &= m + b \\ \Longrightarrow & \frac{p\rho_w}{\rho_b} \times b &= m + b \\ \Longrightarrow & \frac{p\rho_w}{\rho_b} \times b - b &= m \\ \Longrightarrow & \left(\frac{p\rho_w}{\rho_b} - 1\right) \times b &= m \\ \Longrightarrow & b &= \frac{m}{\frac{p\rho_w}{\rho_b} - 1} \end{aligned}$$

Substituting in the values and rounding off to two decimal places, we have:

$$\begin{aligned} b &= \frac{100}{\frac{0.6 \times 1000.0}{18} - 1} \\ &= 3.09\text{kg} \end{aligned}$$

Thus, Ben will need at least 3.09kg of foam to support his weight if he wants to ensure that at least 40% of it remains above water.

- **BUSINESS AND FINANCE**

  1.

$$\begin{aligned} FV &= A\left(\frac{(1+r)^n - 1}{r}\right) \\ &= 330\left(\frac{\left(1 + \frac{0.091}{26}\right)^{26\times 11} - 1}{\frac{0.091}{26}}\right) \\ &= \$161,817.88 \end{aligned}$$

2.

$$
\begin{aligned}
PV &= A\left(\frac{1-(1+r)^{-n}}{r}\right) \\
\Rightarrow 465700 &= A\left(\frac{1-(1+0.0804)^{-9}}{0.0804}\right) \\
\Rightarrow 465700 &= A \times 6.2365 \\
\Rightarrow \frac{465700}{6.2365} &= A \\
\Rightarrow A &= \$74,673.17
\end{aligned}
$$

3.

$$
\begin{aligned}
FV &= A\left(\frac{(1+r)^n-1}{r}\right) \\
\Rightarrow 800 &= 300\left(\frac{(1.0832)^n-1}{0.0832}\right) \\
\Rightarrow \frac{800}{300} &= \left(\frac{(1.0832)^n-1}{0.0832}\right) \\
\Rightarrow \frac{800 \times 0.0832}{300} &= (1.0832)^n - 1 \\
\Rightarrow \frac{800 \times 0.0832}{300} + 1 &= (1.0832)^n \\
\Rightarrow \ln\left(\frac{66.56}{300}+1\right) &= n \ln(1.0832) \\
\Rightarrow n &= \frac{\ln\left(\frac{66.56}{300}+1\right)}{\ln(1.0832)} \\
&= 2.51\,\text{years}
\end{aligned}
$$

4. *Since interest is 11% pa compounded quarterly:*

$$
\begin{aligned}
A &= \frac{178 \times 12}{4} = \$534 \\
r &= \frac{11}{4} = 2.75\%
\end{aligned}
$$

So:

$$
\begin{aligned}
PV &= A\left(\frac{1-(1+r)^{-n}}{r}\right) \\
&= 534\left(\frac{1-(1+0.0275)^{-7\times 4}}{0.0275}\right) \\
&= \$10,333.34
\end{aligned}
$$

5.

$$
\begin{aligned}
EAR &= \left(1+\frac{r_{nom}}{m}\right)^m - 1 \\
&= \left(1+\frac{0.10}{12}\right)^{12} - 1 \\
&= 10.47\%
\end{aligned}
$$

6. As dividends are being retained for reinvestment, the firms earnings should be growing. Hence we will use the Gordon Growth model to value *Beta Ltd*'s equity.

First we determine the firms growth rate:

$$\begin{aligned} g &= \text{reinvestment rate} \times \text{ ROE} \\ &= 0.50 \times 0.08 \\ &= 0.04 \end{aligned}$$

Next we calculate the equity value:

$$\begin{aligned} P_0 &= \frac{d_1}{r_e - g} \\ &= \frac{d_0 \times (1+g)}{r_e - g} \\ &= \frac{0.10 \times (1+0.04)}{0.08 - 0.04} \\ &= \$2.60 \text{ per share.} \end{aligned}$$

7. We can still use the Gordon Growth model to value *Gamma Ltd*'s shares when dividends are constant. The growth rate we use is 0 and the model simplifies to:

$$P_0 = \frac{d}{r_e}$$

Now the required return on equity is:

$$\begin{aligned} 0.50 &= \frac{0.10}{r_e} \\ r_e &= \frac{0.10}{0.50} \\ &= 20\% \text{ per annum.} \end{aligned}$$

8. First we calculate *Beta Ltd*'s share price from the forward looking P/E ratio and the expected EPS:

$$\begin{aligned} P_0 &= P/E \text{ ratio (forward looking) } \times EPS_1 \\ &= 15 \times 1.10 \\ &= \$16.50 \text{ per share.} \end{aligned}$$

Once we know the share price and constant growth is assumed, we can calculate the growth rate g using the Gordon Growth model:

$$\begin{aligned} P_0 &= \frac{d_1}{r_e - g} \\ P_0 &= \frac{EPS_1 \times DPR}{r_e - g} \\ 16.50 &= \frac{1.10 \times 0.20}{0.09 - g} \\ 0.09 - g &= 0.0133 \\ g &= 7.67\% \text{ per annum.} \end{aligned}$$

9. The utility of this level of wealth is

$$U(W) = \frac{1,010,000^{1-0.2}}{1-0.2} = 79,500.$$

10. As the coin is fair, you have 50% chance of winning $381,000 and 50% of losing $381,000. Therefore, the expected payoff is:

$$\mathbb{E}(\text{payoff}) = 0.50 \times 381,000 + 0.50 \times (-381,000) = 0.$$

The utility of your wealth if you win the game is:

$$U(W) = 1.3 \times (700,000 + 381,000) = 1,405,300;$$

and the utility of your wealth if you lose the game is:

$$U(W) = 1.3 \times (700,000 - 381,000) = 414,700.$$

Therefore, the expected utility of your wealth after the game is:

$$\mathbb{E}(U) = 0.50 \times 1,405,300 + 0.50 \times 414,700 = 910,000.$$

11. We first calculate the expected payoff. The probability of winning $36,000 is 50% and the probability of losing $36,000 is 50%, as it is a fair coin.

$$\mathbb{E}(\text{payoff}) = 0.50 \times 36,000 + 0.50 \times (-36,000) = 0.$$

As the expected payoff is equal to 0, we then calculate the expected utility of your wealth after the game. The utility of your wealth if you win the game is:

$$U(W) = (55,000 + 36,000)^{0.7} = 2,960.25;$$

and the utility of your wealth if you lose the game is:

$$U(W) = (55,000 - 36,000)^{0.7} = 988.84.$$

Therefore, the expected utility of your wealth after the game is:

$$\mathbb{E}(U) = 0.50 \times 2,960.25 + 0.50 \times 988.84 = 1,974.55.$$

Finally, we compare this expected utility of your wealth after the game with the utility of your current wealth, which is

$$U(W) = 55,000^{0.7} = 2,080.91.$$

As the expected utility of your wealth after the game is lower than the utility of your current wealth, you will choose not to play the game.

12. If you take the insurance, you need to pay $1,500 so your wealth drops to $39,500 with 100% certainty. This gives utility of:

$$\log_e(39,500 + 100) = 10.59$$

If you don't take the insurance, your wealth at the end of the year will be 0, $20,500 or $41,000 with probability 0.04, 0.06 and 0.9, respectively. Therefore, the expected utility is:

$$\begin{aligned} \mathbb{E}(U) &= 0.04 \times \log_e(0 + 100) + 0.06 \times \log_e(20,500 + 100) + 0.9 \times \log_e(41,000 + 100) \\ &= 10.34 < 10.59 \end{aligned}$$

You are better off, in the sense of having a higher expected utility, with insurance. Therefore, you should take out insurance at this rate. Note: your answers might be slightly different to the ones above due to roundings.

13. If you don't take the insurance, your wealth at the end of the year will be 0, \$4,500 or \$9,000 with probability 0.04, 0.06 and 0.9, respectively. Therefore, the expected utility is:

$$\begin{aligned}\mathbb{E}(U) &= 0.04 \times \log_e(0+500) + 0.06 \times \log_e(4,500+500) + 0.9 \times \log_e(9,000+500) \\ &= 9.00\end{aligned}$$

Let x be the maximum amount of premium you are willing to pay, then your wealth after insurance is certain to be \$9,000 - x and the utility of your wealth should be equal to 9.00.

$$\begin{aligned}\log_e(9,000-x+500) &= 9.00 \\ 9,500-x &= e^{9.00} \\ x &= \$1,396.92\end{aligned}$$

Therefore, you are willing to pay up to \$1,396.92 for this insurance.

- **ARITHMETICS**

1. 9 and 6

2. 5

3. 60 seconds

4. No

5. (**1**) $2 \times 2 \times 2 \times 3 \times 3$
(**2**) 2×13
(**3**) No
(**4**) $2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 13$

6. $10\sqrt{2}$

7. 0

8. $3\frac{9}{32}$

9. $\frac{83}{110}$

10. 0

11. $1\frac{7}{43}$

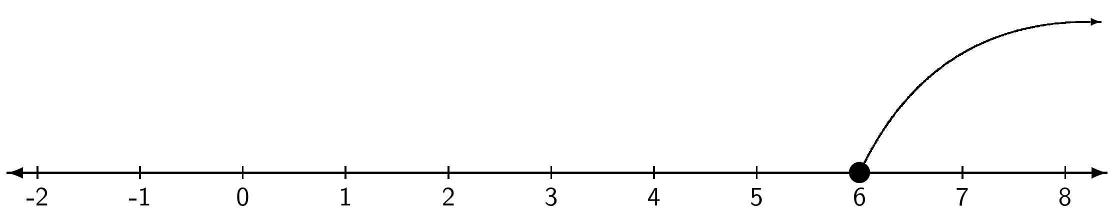
12. 9

13. 7

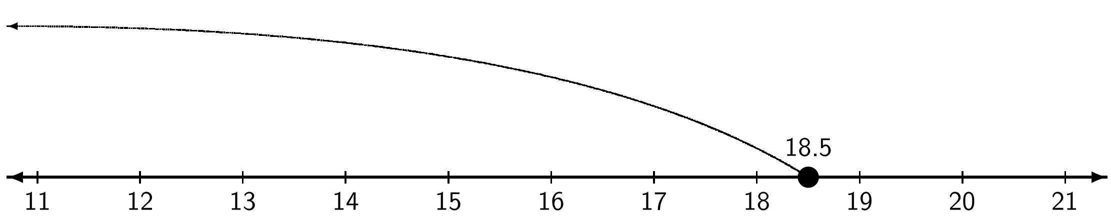
- **ALGEBRA**

1. $z = 5$

2. In interval form the answer is $[6.0, \infty)$ and on a real line the answer is:



3. In inequality form the answer is $x \leq 18.5$ and on a real line the answer is:



4. $y = 1$

5. $y = -\frac{1}{4}$

6. $z = \frac{3}{2}$

7. $-5z + 20z^2$

8. $4z^2 + 4z + 1$

9. $z = \frac{5}{2}$ and $z = -\frac{7}{2}$

10. 24

11. $z = -\frac{1}{2}$

12. $\sqrt{10} + \sqrt{15}$

13. $x = -2$

14. $z = 1$

15. $x{=}-\frac{3}{14}$

16. $x^{-5}y^{-2}$

17. $x \geq 4$

18. $15x^3$

19. 8 and 9

20. 6 and 11

- **SIGMA NOTATION**

1. $z = -4$

2. $z = 2$

3. 272

4. $-72z$

5. 216

6. $\sum_{k=4}^{9} \frac{-1}{k}$

7. $z = 3$

- **SETS**

1. $\{5, 6\}$

2. (**1**) $\{0, 9\}$
(**2**) $\{-3, 3, 5, 2, 0, 9, 8, 6, 1\}$
(**3**) $\{-3\}$
(**4**) $\{3, 5, 2, 8, 1, 6\}$

3. (**1**) $\{-1, 0, 4, 8, 6, 1\}$
(**2**) $\{-1, 5, 2, 0, 4, 8, 6, 1\}$
(**3**) $\{-1, 4, 6\}$
(**4**) $\{5, 2\}$
(**5**) $\{3, 7, 9\}$
(**6**) $\{3, 7, 9\}$
(**7**) $\{5, -1, 2, 4, 8, 6\}$
(**8**) $\{5, 2, 8\}$
(**9**) $\{3, -1, 7, 0, 9, 4, 8, 1, 6\}$

4. (1) $\frac{2}{5}$
   (2) $\frac{1}{5}$
   (3) $\frac{2}{5}$
   (4) $\frac{1}{5}$
   (5) $\frac{3}{5}$
   (6) $\frac{1}{2}$
   (7) $\frac{1}{5}$
   (8) $\frac{7}{10}$
   (9) $\frac{2}{5}$

- **STRAIGHT LINES AND SIMULTANEOUS EQUATIONS**

1. $\sqrt{377}$
2. $m = -\frac{10}{7}$, $c = -\frac{1}{7}$
3. $m = \frac{5}{2}$, $c = -\frac{5}{4}$
4. $y = 2x - 9$
5. $y = -\frac{7}{16}x + \frac{63}{16}$
6. $y = -3x - 2$
7. $y = 4x + 8$
8. $y = -\frac{1}{3}x + 10$
9. Yes
10. $x = -2$
11. $y = 0$
12. $x = 0$
13. $y = 8$
14. $(-9, 56)$
15. There is no solution; the lines are parallel.
16. $x = \frac{\pi}{4}; \frac{5\pi}{4}; \quad y = -2$
17. The lines intersect at the point $(4, -5)$
18. The lines do not interset, they are parallel.

- **FUNCTIONS, QUADRATICS AND TRIGONOMETRIC FUNCTIONS**

1. (1) N
   (2) L
   (3) I
   (4) G
   (5) F

(**6**) S
(**7**) K
(**8**) J

2. $x \in (-\infty, 0) \cup (0, \infty)$

3. $w \in (-\infty, \infty)$

4. $(0, \infty)$

5. $(-\infty, 0) \cup (0, \infty)$

6. The domain is $(-\infty, 8) \cup (8, \infty)$; the range is $(0, \infty)$

7. $y = 4$ or -1

8. (**1**) $z = 0$ or $z = \dfrac{8}{9}$

(**2**) $z = \dfrac{5}{2}$ or $z = -\dfrac{4}{7}$

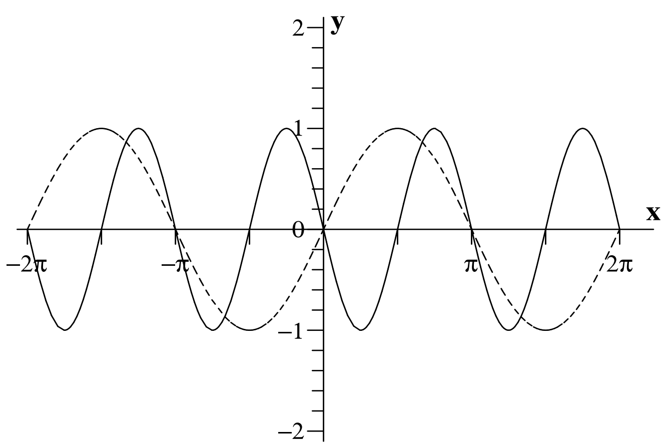
(**3**) $x = -\dfrac{5}{4}$ or $x = 9$

(**4**) $y = -\dfrac{8}{3}$

9. $z = \dfrac{1}{2}$

10. 49

11. The graph of $y = \sin x$ is dashed; the graph of $y_1 = -\sin(2x)$ is solid.



12.

$-144°\quad -225°\quad 63°\quad -345°\quad -27°\quad 345°\quad -189°\quad -660°$

13.

$-\dfrac{10\pi}{9}\quad -\dfrac{21\pi}{10}\quad \pi\quad \dfrac{3\pi}{4}\quad -\dfrac{6\pi}{5}\quad -\pi\quad -\dfrac{5\pi}{3}\quad -6\pi$

• LOGARITHMS AND EXPONENTIALS

1. (**1**) 12
(**2**) 1
(**3**) −3
(**4**) 1
(**5**) −6

(**6**) 0
(**7**) -3
(**8**) $\frac{1}{2}$

2. 902.00

3. 74.08

4. (**1**) \$428.00
(**2**) \$428.49
(**3**) \$428.74
(**4**) \$428.92
(**5**) \$429.00

5. 0.018

6. 11.5525

7. \$456.97

8. \$1521.96

9. 19

10. \$379.71

- **DERIVATIVES AND INTEGRATION**

1. $y' = 0$

2. $y' = 6x$

3. $y' = -42x^5$

4. $y' = \sin x$

5. $y' = 6e^x$

6. $y' = \frac{8}{x^2} + 4\cos x - \frac{7}{2\sqrt{x}} + 2\cos x$

7. Q1 $f'(x) = -3x^2 - 6x + 9$
Q2 $x = -3$ or 1
Q3 $f''(x) = -6x - 6$
Q4 $f'(5) = -96$

8. $\frac{dy}{dx} = \frac{126}{x^3\,(10 + 9x^{-2})^8}$

9. $\frac{114}{(4r + 10)^2}$

10. $\frac{117x^2 + 110x - 63}{(-1 + 6x^2 - 9x)^2}$

11. $\frac{-60t^2 - 54t - 90}{(3t^2 + 10t)^2}$

12. $192t^3 - 54t^2 - 24$

13. $2x^5 - \frac{7}{2}x^2 + 5x + C$

14. 123

15. $\int_{3\pi/2}^{5\pi/2} (-3\sin(-3x))\,dx = 0$

- **MATRICES**

1. $\begin{pmatrix} -6 & 0 & -6 \\ 11 & 5 & -9 \end{pmatrix}$

2. $\begin{pmatrix} 3 & -4 \\ -3 & 0 \\ -7 & 1 \end{pmatrix}$

3. $\begin{pmatrix} 40 & 29 \\ 22 & 13 \end{pmatrix}$

4. $\begin{pmatrix} -1/9 & -1/9 \\ 2/3 & -1/3 \end{pmatrix}$

5. $\begin{pmatrix} -18 & -18 & 36 \end{pmatrix}$

6. $\begin{pmatrix} 11 \\ 5 \\ -3 \end{pmatrix}$

7. (b) The solution to the simultaneous equations is $x = 76$, $y = -10$ and $z = -41$.

- **BASIC PROGRAMMING WITH PYTHON**

1. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

(**1**) 262144
(**2**) -2
(**3**) -138
(**4**) 10077696
(**5**) 726
(**6**) 456
(**7**) 1271
(**8**) -26
(**9**) -32
(**10**) 16

2. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

The distance between the two points is 8.06225774829855.

3. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

The value calculated is 1.0 as required.

4. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

The values calculated are 0.0697564737441253, 0.9975640502598242 and 0.06992681194351041.

5. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

Both expressions evaluate to 12.0

6. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

The calculated value is 0.049787068367863944.

7. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

$$B = \begin{pmatrix} -0.9230769230769231 & 0.07692307692307693 \\ 1 & 0 \end{pmatrix}$$
$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$BA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

We can see that – allowing for rounding errors – AB and BA are both approximately equal to the identity matrix.

8. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.

$$x = -0.4130434782608696$$
$$y = 2.043478260869565$$

9. Please note that there may be some slight rounding errors present, and that the results may not be formatted exactly as you would find in Python.
The values printed would be:
-1, -8, -27, -64, -125, -216, -343, -512, -729, -1000, -1331, -1728, -2197, -2744, -3375, -4096, -4913, -5832, -6859 and -8000

- **SCIENCE**

1. The initial population vector P_0, the Leslie matrix L, and the final population vector P_3 are:

$$P_0 = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 3 & 8 \\ 0.4 & 0 \end{pmatrix} \quad \text{and} \quad P_3 = \begin{pmatrix} 950 \\ 96.8 \end{pmatrix}.$$

2. 7.5L
3. 46.62 kPa
4. 44.4kPa
5. (**1**) 1.83 kg
(**2**) 3.09 kg

- **BUSINESS AND FINANCE**

1. *$161,817.88*
2. *A. $74,673.17*
3. *A. 2.51*
4. *A. $10,333.34*
5. 10.47%
6. $2.60
7. 20% per annum
8. 7.67% per annum
9. 79,500
10. 0; 910,000
11. no
12. yes
13. 1,396.92